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# Class Discussion After Solving a Fermi Problem in Groups: Analysis of Teacher and Student Actions

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**ABSTRACT**

This study analyses the class discussion that follows students working on a Fermi problem in groups, with a particular focus on the interactions between teachers and students and their influence on mathematical reasoning. The research involved a sample of 24 fifth-grade students from a public school in Mollet del Vallès, Spain, who were tasked with solving the following question: "How many people can fit in the schoolyard?" The teacher actions (*redirecting*, *progressing*, *focusing*) and student interventions (*explanations*, *initiatives*, *responses*). The findings demonstrate the crucial role of the teacher in orchestrating the discussion, which predominantly followed the initiation-response-evaluation (IRE) pattern. The implementation of key teaching actions, such as *recap* and *demonstration*, enabled students to reflect upon and refine their own approaches. While the students' participation was largely reactive, their contributions were crucial for the progression of the discussion. The findings of this study emphasize the significance of well-structured classroom discussions in prompting students to engage in critical reflection on their chosen solutions and methodologies.

**KEYWORDS**

Class discussion; Fermi problem; mathematical modeling; teacher actions

## Introduction

In the current educational landscape, there is a growing emphasis on incorporating open-ended tasks that foster mathematical modeling. The objective is to implement activities at various educational levels that facilitate students' ability to connect mathematical concepts with their surrounding environment (Vorhölter et al., 2014). Fermi problems are a clear example of a modeling activity that can be effectively addressed in the context of primary education (Albarracín & Gorgorió, 2014). These are open, nonstandard problems that require students to make assumptions about a situation and estimate relevant quantities before performing simple calculations (Ärlebäck, 2009; Efthimiou & Llewellyn, 2007). Originally used by physicist Enrico Fermi to illustrate the power of deductive reasoning and support laboratory work, the approach has since been applied across multiple disciplines (Ärlebäck & Albarracín, 2019). The method involves decomposing complex problems into smaller, manageable components, estimating values for each part, and combining these estimates into a coherent solution (Carlson, 1997). In classroom practice, activities such as guesstimation, searching for data, collecting and analyzing data, and taking measurements help students work through Fermi problems, linking them closely to the broader process of mathematical modeling (Albarracín & Ärlebäck, 2019; Ferrando & Albarracín, 2021; Peter-Koop, 2009).

As with Fermi problems, a multitude of approaches can be employed to arrive at a solution to a mathematical problem. When students engage with a problem, they frequently develop a range of ideas and methods (Kooloos et al., 2020). Consequently, educators must structure their lessons so that students can rely on one another to solve problems and disseminate their strategies and solutions

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during classroom discussions (Stein et al., 2008). In this context, the teacher assumes a pivotal role, orchestrating the discourse through techniques such as facilitating turn-taking, posing questions, and offering constructive feedback (Smith & Stein, 2011). The objective is to foster mathematical reasoning and meaningful learning, enabling students to engage actively in the discourse by articulating and substantiating their perspectives (Drageset, 2015a). Developing a solid understanding of mathematics entails the capacity to share solutions, propose conjectures, engage in discourse concerning the representation of problems, explain the means of solution, demonstrate the viability of solutions, and identify overarching patterns (Franke et al., 2007). Consequently, it is imperative to cultivate such scenarios within the classroom.

The existing literature on discussion in the mathematics classroom predominantly concentrates on the examination of the teacher's role and its impact on student engagement (Cazden, 2001; Franke et al., 2007). Conversely, numerous studies on modeling activities concentrate on the way students construct models to solve a given problem (Albarracín & Gorgorió, 2019; Haberzettl et al., 2018). Some studies have established a connection between mathematical modeling and class discussions, delineating the teacher's role in the context of structured argumentation (Solar et al., 2022). However, there is a lack of research on class discussions specifically involving Fermi problems in primary education, a gap this study aims to address.

The students' problem-solving process was based on four stages. Initially, students worked on the problem individually. Subsequently, in groups, they decided on a solution plan. Thereafter, they tested their solution plans. Finally, all groups shared the strategies used and results obtained through a whole-class discussion. In a previous study, we identified the potential of the small group work (the second stage) when creating models to solve a Fermi problem, as well as the individual roles of each student within the group (Brunet-Biarnes & Albarracín, 2024). The present study focuses on the final whole-class discussion, during which students share their solutions, explain their problem-solving methods, and justify their answers. The discussion was understood as a space where ideas emerging from group work could be taken up and developed collectively, depending on how the classroom interaction unfolds. The research objectives are *Objective 1*: to characterize teacher actions and students' interventions; and *Objective 2*: to identify patterns between teacher and student interactions. The focus is placed on how students establish connections between mathematical concepts and procedures to construct a viable model, rather than on the linguistic form of their communication. Consequently, the classroom discussion is examined as a collaborative environment for contrasting, refining, and validating group ideas. By analyzing this final phase, the study aims to gain insight into how teacher-guided orchestration facilitates the emergence of mathematical reasoning and the consolidation of modeling strategies at the primary education level.

## Theoretical Framework

This section is divided into three parts: mathematical modeling, Fermi problems and class discussions.

### *Mathematical Modelling*

Research on teaching mathematics using modeling has expanded in recent years, both in terms of the objectives pursued and the methodologies employed (Abassian et al., 2020). Some studies concentrate on the final models produced for specific activities; however, these frequently neglect to examine the modeling processes employed in their construction (Yanagimoto & Yoshimura, 2013). Concurrently, other research examines the manner in which students construct mathematical models to solve problems, with disparate perspectives (Borromeo Ferri, 2006; Hankeln, 2020).

In the context of mathematics education, there is a general consensus that modeling processes are cyclical (Blum & Leiss, 2006; Galbraith & Stillman, 2006; Kaiser & Stender, 2013; Maaß, 2006). When solving a problem, students move through a series of phases that alternate between two domains: real-world context and mathematics. They continuously reevaluate the situation and

adjust their models accordingly, thereby refining both their models and solutions through repeated cycles. These models are then adapted to meet the specific demands of the problem they are solving (Blum & Borromeo Ferri, 2009). A key aspect of the process also involves students communicating their reasoning.

The definition proposed by Lesh and Harel (2003) is adopted here, whereby mathematical models are defined as conceptual systems that employ a variety of representational media to construct, describe, or explain other systems. A model comprises a conceptual system for describing or explaining the relevant mathematical objects, relations, actions, patterns, and regularities, together with the procedures that enable useful constructions, manipulations, or predictions. Considering this definition, a model can be understood as a means of representing reality using diverse languages, including words, drawings, and schemes, which encompass both conceptual and procedural elements. The objective is to describe a system, typically a complex one, in a way that students can comprehend. These models facilitate the description and understanding of complex systems, particularly in problem-solving contexts, where they assist in representing situations and suggesting solutions based on mathematical reasoning. This approach is particularly beneficial in primary education, where students may lack advanced mathematical tools but can still represent ideas through drawings or everyday language (Albarracín & Gorgorió, 2019).

## *Fermi Problems*

Fermi problems are a clear example of a modeling activity that can be effectively addressed in the context of primary education (Albarracín & Gorgorió, 2014). They are defined as open-ended questions that provide little or no information to support their solution (Ärlebäck, 2009; Efthimiou & Llewellyn, 2007). This defining lack of given data makes estimation a central component of students' work, encouraging learners to produce plausible numerical assumptions and to justify them. Estimation renders the problem accessible to primary school students, who may not yet have formalized methods for obtaining exact results. In this way, students can meaningfully engage with complex situations through simplified representations and approximate reasoning, rather than being constrained by the need for precise information. As a consequence of this feature, primary school students are able to devise a range of models to address these questions (Albarracín & Gorgorió, 2019). Fermi problems admit multiple solution paths and can be approached through diverse strategies, and this availability of different approaches not only underscores the modeling character of these tasks but also creates productive opportunities for comparing and evaluating alternative assumptions, representations, and calculations (Albarracín & Gorgorió, 2014).

Fermi problems frequently relate to real-life scenarios with which students are familiar, such as estimating the number of cars in a traffic jam (Peter-Koop, 2009). These problems facilitate the modeling of a real-world phenomenon while simultaneously developing estimation skills (Ärlebäck & Albarracín, 2019). For example, Peter-Koop (2009) examined how primary school students (ages 10–12) approached these problems and discovered that they employed a range of strategies, developed new mathematical knowledge, and followed a multicyclic modeling process. Furthermore, recent studies with trainee teachers emphasize the significance of flexibility (the capacity to switch between strategies) and adaptability (the ability to select context-appropriate strategies) for the effective resolution of Fermi problems (Segura & Ferrando, 2023a, 2023b).

In a study conducted by Albarracín and Gorgorió (2019), students between the ages of 11 and 12 were observed as they constructed models to solve Fermi problems. The findings revealed the existence of diverse strategies and methods employed by the students in their model-building processes, each of which represented a unique approach to solving a Fermi problem. Henze and Fritzlar (2009) investigated the impact of group dynamics on the resolution of Fermi problems, noting that these tasks encourage sustained engagement of students throughout the solving process. Haberzettl et al. (2018) explored the potential of Fermi problems to enhance children's modeling abilities, documenting a sequence of lessons that provided a detailed view of implementing modeling tasks in classrooms.

## Class Discussion

Class discussions are an indispensable tool for fostering mathematical reasoning and meaningful learning. In this manner, students are not only able to share their ideas, but also to defend and refine them based on feedback from their peers and their teachers (Drageset, 2015b). For students to develop a deeper understanding of mathematics, it is essential that they can present their solutions, propose conjectures, explore different mathematical representations, explain their problem-solving processes, justify their solutions and make explicit generalizations (Franke et al., 2007, p. 230).

Several broader traditions in mathematics education provide valuable perspectives on classroom interactions. Radford's (2010) semiotic-cultural theory emphasizes embodied and artifact-mediated activity, highlighting how gestures, signs, and tools support learning. Sfard's (2007) communicational framework views mathematics as discourse, focusing on participation in language-mediated practices and the development of shared mathematical meanings. Krummheuer's (2015) dialogical model stresses the social construction of knowledge through dialogue and collective negotiation. Table 1 shows the main distinctions among these approaches. While each approach contributes with important insights into classroom interactions, they do not specifically address the teacher's role in guiding discussions to deepen student reasoning.

Several studies have examined the patterns of classroom discussion in the field of mathematics education, with a particular focus on the interaction between students and teachers. In many instances, discussions are led by the teacher, frequently following the IRE (initiation-response-evaluation) pattern, whereby the teacher poses a question, students respond, and the teacher evaluates the response (Cazden, 2001). Although the IRE pattern may, on occasion, constrain student engagement and focus primarily on procedural knowledge (Franke et al., 2007), other researchers have underscored its adaptability (Wells, 1993). Classroom settings that are more interactive, such as those based on inquiry-based learning and strategy-sharing, facilitate student reasoning and justification (Wood et al., 2006). However, merely increasing student participation is insufficient; teachers must also guide discussions in a way that makes student thinking explicit and encourages them to develop a more advanced mathematical understanding (Franke et al., 2007).

As observed by Stein et al. (2008), the initial attempts at facilitating mathematical discussions frequently lacked the necessary guidance from teachers, resulting in superficial exchanges. This emphasizes the necessity of teacher intervention, not only to encourage participation, but also to facilitate the exploration of more complex mathematical concepts. Brendefur and Frykholm (2000) introduced the concept of reflective communication, whereby discussions are shaped by building on students' ideas, as opposed to instructive communication, which aims to direct students' thinking.

There is a growing consensus that student contributions should play an important role in classroom discourse. However, there is still a lack of consensus on how to achieve this. In order to provide students with this opportunity, classrooms must be designed in a way that allows students to take the lead in the discussion, thereby enabling them to express their ideas and reasoning (Drageset, 2014). Smith and Stein (2011) put forward five key practices that can assist teachers in managing classroom discussions in a more effective manner. These practices are intended to anticipate potential classroom dynamics and provide teachers with the necessary time to make well-informed decisions. The five practices include: *anticipating* possible student responses to complex mathematical tasks, *monitoring* how students respond to tasks (particularly in pairs or small groups), *selecting* specific students to present their work during discussions,

**Table 1.** Classroom interaction perspectives in mathematics education.

| Author | View of classroom interaction | Teacher and student roles |
|---|---|---|
| Radford (2010) | Interaction as a semiotic process where meaning emerges through speech, gestures, and symbols. | Teacher mediates signs; students co-construct meaning. |
| Sfard (2007) | Interaction as participation in mathematical discourse with specific rules. | Teacher embodies authoritative discourse; students adapt their reasoning and language. |
| Krummheuer (2015) | Interaction as socially organized argumentation, negotiated moment by moment. | Teacher and students are co-participants shaping the flow of interaction. |

*sequencing* student responses in a deliberate order, and *connecting* student responses to key mathematical concepts.

One of the most significant challenges for educators in the context of classroom discourse is the decision-making process regarding the selection and prioritization of contributions, and the exclusion of certain elements. This balancing act frequently entails the management of various factors, including lesson objectives, time constraints, and other considerations (Drageset, 2014). Stockero and Van Zoest (2013) introduced the concept of "pivotal teaching moments" (PTMs) as a means of analyzing how teachers recognize and respond to key moments in student thinking. Three productive responses were identified (extend, pursue, emphasize) and two less effective ones (ignore/dismiss, acknowledge but move on). It was established that missing these PTMs can have a negative effect on student learning, which underlines the importance of teachers identifying and acting on these opportunities. In our research, these PTMs are those that have the potential to facilitate significant breakthroughs in students' reasoning. These moments encourage students to reassess the strategies they have employed, to validate the results they have obtained and to improve the methods they use. Furthermore, Smith and Stein (2011) emphasize the significance of teachers' questioning during discussions. Well-crafted questions have the potential to draw attention to aspects of a problem that may have been overlooked, or to open new perspectives. They can also encourage students to clarify their thought processes and make their reasoning accessible to others. This approach prioritizes students' thinking and comprehension of mathematical content, providing teachers with valuable insights into students' ideas, which is crucial for effective teaching and learning (Drageset, 2014; Franke et al., 2007).

Additionally, research conducted by Drageset (2015a) illustrates the significance of student interventions during classroom discussions and their influence on the overall discourse. By analyzing how teacher responses adhere to specific types of student interventions (such as explanations, initiatives, or responses that remain unexplained), Drageset (2015b) identifies patterns where teacher actions either facilitate or impede the development of deeper mathematical reasoning. For instance, teacher-led responses frequently entail the decomposition of tasks into discrete components, which can facilitate participation but may restrict students' capacity to engage with more sophisticated mathematical reasoning. In contrast, directing students' attention through techniques such as requesting justifications or highlighting pivotal elements encourages them to articulate and clarify their reasoning, thereby fostering a more reflective and conceptually rich discourse. This comprehensive examination highlights the necessity for educators to respond to student contributions in ways that extend beyond simple correctness, facilitating a more profound examination of mathematical concepts.

Although there is a substantial body of research examining the role of teachers and students in mathematics classroom discussions, there is a limited evidence base exploring this practice in the specific context of discussions where students present solution strategies for Fermi problems. This study addresses this gap by offering new insights into the intersection of mathematical modeling and class discussions through the process of solving a Fermi problem. In doing so, it contributes to a deeper understanding of how students' mathematical reasoning and engagement unfold in these unique problem-solving scenarios.

## Methodology

This qualitative study is situated within an interpretivist perspective, specifically drawing on the framework of symbolic interactionism (Blumer, 1986), which posits that mathematical learning emerges from social interaction and classroom discourse. The teacher acts as an orchestrator of discourse (Smith & Stein, 2011) who guides student reasoning through intentional actions.

The data for this study were gathered from a sample of 24 fifth-grade students at a public school in Mollet del Vallès (Barcelona, Spain) and the class teacher. Two sessions, each lasting an hour and a half, were both video and audio recorded. All participants were informed of the purpose of the research and its methods. A Consent to Publish was obtained from research participants. The students were aged between ten and eleven. This convenience sample was selected because the teacher had prior

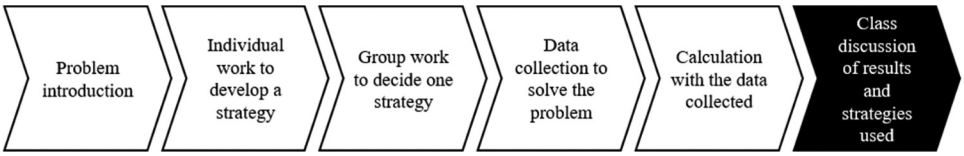



**Figure 1.** Stages in the students' problem-solving process.

experience conducting research on Fermi problems, which made him well-versed in handling this type of problem and familiar with the data collection process employed in the study.

The Fermi problem posed was: *"How many people can fit in the school yard?"* Students had to estimate how many individuals could be accommodated in a specific area. This problem, previously studied by Albarracín and Gorgorió (2014, 2019), allowed us to anticipate students' solving strategies and mathematical models. Considering that the area was familiar to them, students could better experiment, measure, visualize, and understand the situation. To contextualize it, the problem was framed around organizing a year-end party, aiming to determine how many guests the playground could host. Given the school's experience with similar events, this scenario was relevant, understandable, and engaging for the students.

The activity took place during a class period, comprising two sessions of one and a half hours each. The structure followed to carry out the data collection was as follows:

This structure (Figure 1) combined individual work, group collaboration, and whole-class discussion. In the first session, the problem was presented to the whole class, allowing each student to think independently and propose a solution strategy. Students were then divided into heterogeneous groups of four to discuss their strategies and agree on an action plan. They collected missing data on the playground and performed the necessary calculations. In the second session, each group presented their strategies and solutions. Finally, students completed a questionnaire indicating whether they would change their initial strategy. This paper will focus on the whole-class discussion that occurred after a Fermi problem was solved in groups. The discussion was led by one teacher who guided the conversation.

## Data Analysis

Once the data collection process was complete, the initial stage of the analysis involved transcribing and structuring all the recordings of the class discussion. The subsequent analysis of the data was conducted in accordance with a three-step process. The first step was to identify the progression of mathematical ideas. The second step was to characterize the interventions of both the teacher and the students. Finally, the third step was to identify possible patterns during the class discussion.

For the analysis, the students were coded based on gender (male or female), followed by the group number and the student's number within the group. For example, a male student from Group 3 would be coded as M31.

### *Progression of Mathematical Ideas*

The initial phase of the analysis focused on the mathematical concepts discussed during the class, including the models students used to solve the Fermi problem. These models are characterized following the categorization proposed by Albarracín and Gorgorió (2014, 2019). The most relevant for this study were: *reduction and use of proportion*, entails the resolution of a comparable issue with smaller values, accompanied by the application of a proportional factor between the two scenarios; and, g*rid distribution*, is the arrangement of people or objects on a grid, with the quantities of said people or objects estimated based on each dimension (e.g., height and width). Following the methodology outlined by Brunet-Biarnes and Albarracín (2024), an option was included for students who presented strategies that may not be appropriate for a modeling process to solve the given problem. These were classified as "*no strategy identified*."

### *Teacher's and Student's Interventions Characterization*

There are several frameworks that can be used for the analysis of classroom discussions. These include Toulmin's (2003) model, Solar et al. (2022) framework on argumentative orchestration, and the inquiry-cooperation model proposed by Alrø and Skovsmose (2002). The choice of a framework in this study is guided by the need to capture the dynamic, moment-to-moment co-construction of meaning between teacher and students. Although the aforementioned approaches concentrate on argumentation or dialogic interactions, they do not fully encompass the role of the teacher in guiding discussions where students share and compare strategies. Drageset's (2014, 2015b) framework, which focuses on teacher actions following student interventions, provides a more structured approach to analyze how teachers facilitate and deepen class discussions. Unlike other models that focus primarily on argumentation or dialogic exchanges, this framework captures the ways in which teachers actively direct the discussion, making it possible to identify patterns in teacher-student interactions and understand how classroom dialogue unfolds when students share and compare strategies. This makes it the most suitable choice for this study. Similar studies have employed this model, such as Kooloos et al. (2020) who analyzed class discussions concerning diverse solution approaches to a given problem. Gustafsson (2024) also incorporated a few additional subcategories when analyzing a class discussion on multiple-choice tasks. However, prioritizing this interactional lens involves certain analytical trade-offs. While it allows for a systematic and rigorous categorization of teacher-student interactions, it necessarily foregrounds teacher moves and the functional structure of the discourse. Consequently, other dimensions like the in-depth evolution of students' mathematical ideas, the quality of their individual reasoning, or the broader sociocultural dynamics of the classroom remain less explored.

To analyze the data, we applied the pre-defined coding categories outlined by Drageset (2014), which describe teacher actions within the framework of redirecting, progressing, and focusing. These actions refer to specific types of moves the teacher makes in response to students' contributions during a class discussion, and they can be divided into three main categories:

- *Redirecting actions* refer to teacher actions designed to facilitate a shift in students' approach or strategy. The teacher may acknowledge a student's response but redirect their approach through corrections, suggestions, or questions that encourage reconsideration. This category includes *putting aside students' comments*, explicitly *advising students to use an alternative strategy*, and *correcting questions*.
- *Progressing actions* encompass teacher actions aimed at ensuring the class advances toward problem resolution. This may involve demonstrating solutions, simplifying tasks, or breaking the problem into manageable steps. This category includes *demonstration of steps* or the entire solution, *simplifications* of the task providing hints, *closed progress details* by guiding students through the process, and *open progress initiatives* in which the teacher makes comments or asks questions that allow students to explain their reasoning.
- *Focusing actions* aim to direct students' attention to specific elements of a problem or solution process through comments or questions that prompt explanation, justification, or highlighting key details. This category has two subcategories. *Requests for student input* are actions in which the teacher requests that students explain details, justify their answers, apply knowledge to similar problems, or seek validation from peers. This category includes *enlightening details* of the solution process, *justifying* the reasoning behind the methods used, or applying it to similar problems, and *requesting peer-assessment*. The second subcategory is *pointing out* that refers to actions where the teacher underscores or reiterates important information of the problem. This category includes *recap* of the solutions or clarification of key points and *noticing* important details.

Conversely, we analyzed students' interventions using the pre-defined categories proposed by Drageset (2015a). In this framework, student interventions are understood as any contributions or turns that students make during classroom discussions, particularly in mathematics, and they can be classified into several types:

- *Explanations* are comments made by students to elucidate their thought process and reasoning behind a solution. Typically, explanations are requested by the teacher to ensure understanding and facilitate knowledge sharing among the class. This category can be further subdivided into explanations of *reasoning*, of a *procedure* and of a *concept*.
- *Student initiatives* are comments that originate from students and pertain to topics or questions that demonstrate their interest and curiosity. These comments typically deviate from the Initiation-Response-Evaluation (IRE) pattern, as students independently intervene in the discussion. This category can be further subdivided into *pointing out* information, *suggesting* alternative methods, *correcting* other approaches and *asking how or why*.
- *Partial answers* are comments that contain incomplete content, indicating a degree of understanding but requiring further detail or clarification. This category can be further subdivided into *correct but partial comments*, *insufficient* and *incorrect – but correct observation*.
- *Teacher-directed responses* are comments that are guided by the teacher's questions or explanations. They represent a controlled and defined form of interaction, occurring in scenarios where the teacher has broken a complex problem into smaller, simpler steps or has simplified the task by providing additional information. This category can be subdivided into *correct as a response to closed progress details*, *correct as a response to a simplification*, *confirming or rejecting the teacher's suggestion*, *quoting the teacher* and *off-topic responses*.
- *Answers without explanation* are comments wherein students proffer answers without elucidating their thought process, thereby forfeiting the chance for a more profound comprehension. This category can be further classified into correct but *omitting information*, *demonstration*, *incorrect (no apparent reason)*, *incorrect (resulting from closed progress details)*, *incorrect action* and *unable to answer*.

To ensure methodological rigor and transparency, both researchers independently coded the contributions of teachers and students. Out of a total of 211 contributions, the coders agreed on 93% (196) of the instances. Discrepancies were discussed in detail until a consensus was reached. Most disagreements concerned the teacher's actions within the focusing category, particularly distinguishing between "recap" and "notice." This procedure ensured that coding decisions were consistent and reliable, supporting the credibility and reproducibility of the analysis. Subsequently, we present an example of how teacher and student interventions were categorized during a segment of the class discussion following the resolution of a Fermi problem in Table 2.

**Table 2.** Example of the categorization of data in accordance with the framework proposed by Drageset (2014, 2015a).

| Person | Interaction | Categorization |
|---|---|---|
| Student F62 | We lined up, and the last person moved to the front and so on repeatedly. (. . .) On the line outside of the half field, on the side where the bench was, we lined up from the beginning to half the field, and the person at the back moved to the front, back to front, and so on repeatedly, and we counted, and it gave us 38. | Explanation |
| Teacher | Okay, let's see, you explained something that is quite hard to visualize when explained. But visually, it's very easy. F62, can you come with me for a moment and let's do it? | Progressing action (Demonstration) |
| Teacher | Notice that we just did something ourselves, measuring this width of the classroom in people, right? Had this also occurred to you (another group)? But of course, we only measured the width, not the total number of people that fit in the classroom. | Focusing action – Pointing out (Recap) |
| Teacher | Does anyone have any idea about how measuring the width of the classroom in people could help? | Progressing action (Open progress initiative) |

## Results

During the class discussion, each of the six groups presented the strategy that they had employed to solve the problem and their findings. The most frequently employed model was *reduction and use of proportion* (3), followed by *grid distribution* (1 complete, 2 incomplete). Table 3 illustrates the progression of students' models throughout the session, showing their transition from the initial individual model to the final group model presented. To examine whether students reconsidered their strategies after the discussion, we referred to their responses in the post-session questionnaire, in which they indicated whether they would modify their initial approach and provided a justification for their choice. Moreover, as seen in Table 3, four students would alter their strategy, nine students assert that their group strategy is appropriate and would not modify it, one student claims that all strategies are valid, and finally, ten students would not change their strategy and provide no rationale. Due to data limitations, we cannot determine the reasons behind students' decisions to maintain or modify their strategies. Only five students explained their reasoning, while the rest either kept their initial approach or provided no explanation. Thus, "no change" responses may reflect different factors, but the available data does not allow us to distinguish among them.

Additionally, Figure 2 shows the progression of the mathematical ideas introduced by the teacher (red) and by the students (blue) during the class discussion. The primary models discussed were *reduction and use of proportion* and *grid distribution*. At the outset of the discussion, the teacher aimed to give students the opportunity to articulate their reasoning. Through targeted questioning, the teacher gained insight into the strategies students used. As the discussion progressed, strategies and outcomes across groups were compared, shifting the focus to their practical viability and to identifying the most effective approach to solving the problem. The discussion concluded with a brainstorming of similar problems where the same solving process could be applied.

Furthermore, in the course of the class discussion, the teacher spoke for a total of 62.2% of the time, while students participated for 37.8% of the time. The percentages were calculated manually by timing each contribution of the teacher and the students and summing these intervals. Brief pauses during students' explanations were included as part of the students' speaking time, as they

**Table 3.** Progression of models used by students before and after the class discussion.

| | | Individual strategy | Group strategy | After class discussion |
|---|---|---|---|---|
| G1 | M11 | EC | RP | No changes with no explanation given. |
| | M12 | NS | | No changes with no explanation given. |
| | F11 | EC | | Different strategy: Grid distribution |
| | F12 | EC | | No changes with no explanation given. |
| G2 | F21 | GD | GD (incomplete) | Different strategy: Reduction and use of proportion |
| | F22 | EC | | No changes with no explanation given. |
| | F23 | RP | | Different strategy: Reduction and use of proportion |
| | F24 | NS | | No changes with no explanation given. |
| G3 | M31 | RP | RP | Claims group strategy is correct and would not change it. |
| | M32 | RP | | Claims group strategy is correct and would not change it. |
| | F31 | GD | | Claims group strategy is correct and would not change it. |
| | F32 | NS | | No changes with no explanation given. |
| G4 | M41 | RP | GD | Claims group strategy is correct and would not change it. |
| | M42 | RP | | Claims group strategy is correct and would not change it. |
| | F41 | RP | | Claims group strategy is correct and would not change it. |
| | F42 | NS | | Claims all strategies are correct. |
| G5 | M51 | RP | RP | Claims group strategy is correct and would not change it. |
| | M52 | EC | | Claims group strategy is correct and would not change it. |
| | F51 | RP | | Claims group strategy is correct and would not change it. |
| | F52 | RP | | Would consider the space between people. |
| G6 | M61 | EC | GD (incomplete) | No changes with no explanation given. |
| | M62 | RP | | No changes with no explanation given. |
| | F61 | NS | | No changes with no explanation given. |
| | F62 | NS | | No changes with no explanation given. |

Table codes: EC (exhaustive count), RP (reduction and use of proportion), GD (grid distribution), and NS (no strategy identified).




**Figure 2.** Progression of mathematical ideas during the class discussion following the resolution of a Fermi problem in groups.

**Table 4.** Students' interventions identified in the classroom discussion analysis.

| Type of intervention | Amount |
|---|---|
| Student initiatives | 6 (6,90%) |
| Explanations | 21 (24,14%) |
| Partial answers | 7 (8,01%) |
| Teacher-led responses | 51 (58,62%) |
| Unexplained answers | 2 (2,30%) |
| Total | **87** |

were actively thinking or preparing their next utterance. This approach provided a straightforward measurement of the proportion of speaking time for each participant. In total, there were 126 instances of teacher actions, compared to 85 instances of student interventions. The higher number of teacher actions is attributable to the fact that a single student intervention often resulted in multiple teacher responses. These actions are categorized separately, which increases the total count attributed to the teacher during the class discussion, and which also aligns with the proportion of time the teacher spoke.

During the class discussion after solving the Fermi problem in groups only 10 of the 24 students engaged individually to explain their reasoning or results obtained; the remaining 14 students did not participate directly. The students who participated in the discussion were F11, F21, F23, F24, F42, F51, F62, M32, M41 and M52. Among them, six students (F11, F24, M32, F42, F51, F62) assumed the role of group spokespersons, elucidating their group's methodology for solving the problem and presenting the outcomes achieved. Although not all students participated in the class discussion individually, the teacher occasionally posed closed questions to which the majority of students responded. This approach enabled more students to engage in the discussion, even if their contributions were not particularly detailed.

As shown in Table 4, the most frequent student interventions were *teacher-led responses* (58,62%) and *explanations* (24,14%), which students used to clarify their reasoning, illustrate strategies, or apply approaches to other problems. *Student initiatives* were less common (6,90%) but notable. These occurred, for instance, when students proactively explained their problem-solving strategies before the teacher guided the discussion, when they questioned the teacher's method for determining how many individuals fit in a line, or when they proposed alternative strategies different from those presented by their group. Such interventions may reflect students' engagement in the discussion and their motivation to contribute or explain their reasoning.

Table 5 shows that teacher actions were primarily categorized as progressing or focusing actions. The most frequent progressing actions were *closed-progress details* (28,56%), used to guide students step by step through the solution process, typically through brief, closed-ended questions. *Open progress initiatives* (10,32%) encouraged students to explain their reasoning and the process they had followed to reach a solution. In addition, although *demonstrations* (3,97%) were less common, they were crucial for addressing specific aspects of the problem, such as the space occupied by each person, which led F52 to modify her strategy. Regarding focusing actions, *recap* (19,84%) and *notice* (7,15%) were frequently observed. *Recap* actions helped summarize and rephrase students' explanations, while *notice* and *justification* actions highlighted key elements of different problem-solving strategies and supported students' reflection on their reasoning.

## *Patterns Identified During the Class Discussion*

An important feature of the data in this study is that the discussion was dominated by the IRE (initiation-response-evaluation) pattern (Cazden, 2001). The teacher consistently initiates (through *open-ended initiatives* or *closed progress details* actions), students respond (with interventions such as *explanations* or *teacher-led responses*), and the teacher evaluates their

**Table 5.** Teacher actions identified in the classroom discussion analysis.

| Type of action | | Amount |
|---|---|---|
| REDIRECTING ACTIONS | | 10 (7,94%) |
| Putting aside students' comments | | 5 (3,97%) |
| Advising a new strategy | | 2 (1,59%) |
| Correcting questions | | 3 (2,38%) |
| PROGRESSING ACTIONS | | 56 (44,44%) |
| Demonstration | | 5 (3,97%) |
| Simplification | | 2 (1,59%) |
| Closed progress details | | 36 (28,56%) |
| Open progress initiatives | | 13 (10,32%) |
| FOCUSING ACTIONS | | 52 (41,27%) |
| REQUEST STUDENT INPUT | Enlighten details | 3 (2,38%) |
| | Justification | 6 (4,76%) |
| | Apply to similar problems | 5 (3,97%) |
| | Request peer-assessment | 4 (3,17%) |
| POINTING OUT | Recap | 25 (19,84%) |
| | Notice | 9 (7,15%) |
| CLASSROOM MANAGEMENT | | 8 (6,35%) |
| **Total** | | **126** |

comments, often summarizing and highlighting key information. This pattern indicates the teacher controlled the flow of the discussion, ensuring also that students' contributions were understood and evaluated. A closer examination of teacher actions following student interventions provides further insights into how the teacher facilitates and guides student contributions. This analysis revealed that a single student intervention often triggered multiple teacher actions. For example, following a *student initiative*, the teacher undertook several actions: *putting aside students' comments* (2), *closed progress details* (1), *open progress initiatives* (1), *justification* (1), and *classroom management* (1). These observations indicate that teacher responses following a student intervention frequently involved multiple actions, illustrating the variety of responses observed. Based on Drageset (2014, 2015a) framework, three distinct patterns were identified:

### *Open Progress Initiative (Teacher) – Explanation (Student)*

This initial pattern is observed when the teacher initiates the explanation of each group's solution, occurring a total of six times. The teacher requests that the students explain how they solved the problem and what result they obtained. In response, the group's spokesperson provides a detailed description of their group's approach and presents their results. This pattern is followed by the action of *recap* in 3 occasions, 2 times by *notice* actions and in 1 occasion by the action *demonstration*. An example of this pattern is the following excerpt from the class discussion:

- Teacher (*Open progress initiative*): Okay, F51 explain how you have solved the problem and what is your result.
- Student F51 (*Explanation*): Our result was 898, okay? And I'll explain why. We thought about the number of students at this school, which is 359, and then we considered that when the whole school goes out to the playground for some activities, we all go to the court and occupy less than half of it, okay? So, that space we occupy already fits 359 students or people. Then, in the other part of the court, we also put the same number, 359 people, in the same space. And then we thought about the areas we hadn't covered and added people there. (. . .)

### *Explanation (Student) – Recap (Teacher)*

This pattern is observed on 11 occasions during the class discussion. Following a student explanation, the teacher provides a summary and paraphrase of the student's explanation. These two actions are followed by a multitude of other actions, including: *closed-progress details* (3 times), *open progress*

*initiatives*, *simplification* and *notice* (two times each action), and finally, by *demonstration* and *justification* (one time each action). This variety of teacher actions suggests that the teacher is highlighting different aspects of the student's reasoning, focusing on details, or adding new information to further expand on the student's ideas. An example of this pattern is the following excerpt from the class discussion:

- **Student M42 (*Explanation*)**: We have measured all the playground from side to side, and the width, the length has given us 40 meters, and the width has given us 20 meters, we have multiplied them and the outcome was 800. So, what we did … (shows hesitation) after, we divided between two and it gave us 400. And we wrote the result is 800 tight and 400 people without being tight.
- **Teacher (*Recap*)**: Okay, let's see if I got it right, they measured the width and the length of the playground, of the court, right? Why? Because when you multiply the size of one side by the other in a rectangle, you get the area. (…) So, what they have done is calculate how many square meters would be needed to cover the whole playground. And what they thought was that if one person fits per square meter, then you have 800 people fitting in. But they also thought, to be more comfortable, if we give each person 2 $m^2$so they can move around, right? And they ended up with 400 people.

### *Closed Progress Details (Teacher) – Teacher-Led Responses (Student) – Recap (Teacher)*

This pattern is observed on 12 occasions during the class discussion. The teacher employs *closed-progress details* as a means of guiding students toward a specific piece of crucial information or detail. The students adhere to the teacher's guidance and engage in the lesson actively. Upon reaching the key point, the teacher employs a *recap* to verify comprehension and ensure all students are aware of the essential information. It is noteworthy that the initial phase of this pattern, characterized by *closed-progress details* and *teacher-led responses*, may be reiterated on multiple occasions before the teacher initiates the recap action. Following a *teacher-led response*, there are up to 27 student instances of *closed-progress details*, thereby underscoring the reiteration of this pattern throughout the class discussion. An example of this pattern is the following excerpt from the class discussion:

- Teacher (*Closed progress details*): And did you use this afterward to solve the problem? Yes or no?
- Group 5 (*Teacher-led responses*): Yes.
- Teacher (*Closed progress details*): Was it part of this solution?
- Group 5 (*Teacher-led responses*): No.
- **Teacher (*Recap*)**: Look, they have obtained information about how many people lined up can cover a part of the playground, and they did it for the long side and the short side. But they didn't use this information directly. Instead, it helped them remember that there was a day when they had already found a number, okay? So, one thing that this Group 5 did was to gather as much information as possible to see if any of that information would help them find the solution.

## Results Discussion

The class discussion encompassed a multitude of aspects associated with the modeling process. The fact that the problem was set in a familiar context for students facilitated their ability to relate the problem-solving process to the real world. Furthermore, students considered aspects such as the spatial requirements for comfort and the difference between the space occupied by adults and children. These two aspects are also observed in the study conducted by Brunet-Biarnes and Albarracín (2024). Regarding the solving process, the students were able to generate mathematical models to address the Fermi problem posed (Albarracín & Gorgorió, 2018; Peter-Koop, 2009). The two models used were *grid distribution* and *reduction and use of proportion*. These strategies have also

been observed in previous studies, including those conducted by Ferrando and Albarracín (2021) and Brunet-Biarnes and Albarracín (2024).

The discussion encompassed several points regarding the accuracy and efficacy of the strategies proposed by the students. Additionally, the discussion addressed the specifics of the students' strategies, including breaking the problem into smaller components and the illustration of the solution process. These discussions align with and extend the findings of Segura and Ferrando's (2023a, 2023b), which highlight the importance of flexibility (the ability to switch between strategies) in pre-service teacher training for solving Fermi problems. While their research focused on future teachers, our findings suggest that, in this specific context, classroom discussions in primary education may also serve as a platform to foster flexibility in students. This opens the door to future research exploring how such discussions can explicitly develop this skill in younger students.

At the beginning of the class discussion, few strategies had been shared, as only one or two groups had presented their work. The initial focus was therefore on ensuring that each group provided a detailed explanation of their reasoning. As more strategies were shared, they were compared along with their results. By the end of the discussion, the objective was to identify the optimal approach and outcome, and to explore similar problems where the same method could be applied. The potential value of the class discussion, in which strategies are examined and reflected upon, is suggested by the observation that a small number of students considered adjusting their approach based on the arguments shared during the session. Although only four students chose to modify their strategy, these opportunities can be seen as pivotal teaching moments (Stockero & Van Zoest, 2013), as they encouraged students to notice previously overlooked aspects and fostered a deeper understanding of the problem.

Regarding objective 1, as evidenced by the results, the discourse was largely dominated by the teacher, who employed *focusing* (41,27%) and *progressing* actions (44,44%) mainly. The most frequently occurring actions were *recap* (19,84%), *closed progress details* (28,56%) and *open progress initiatives* (10,32%). Actions focused on summarizing and clarifying procedures or concepts presented by students were used with great regularity. This approach aligns with that proposed by Franke et al. (2007), who emphasize the importance of making mathematical thinking explicit to ensure collective understanding. Furthermore, connecting students' ideas and linking them with other approaches follows the key practices outlined by Smith and Stein (2011). In addition to these, the teacher employed several other actions during the class discussion, including *justification* (4,76%) and *noticing* (7,15%). The first action provided students with the opportunity to explain their reasoning, while the second was mainly used to highlight important aspects of the solving process. Although *demonstration* (3,97%) was not the most frequently used action, it had a clear impact on how students reflected on their own strategies. Aspects highlighted during the examples and demonstrations given by the teacher were also used by students to justify modifications in their initial strategies. Some of these aspects were also raised during *recap* actions.

Student participation was limited to ten individuals, six of whom were group spokespersons presenting their collective work. This low engagement likely stems from the activity's design: since groups had already solved the problems internally, the discussion focused on evaluating the strategies and the results obtained. Introducing varied task formats or allocating more time for spontaneous debate could foster broader involvement. Additionally, in terms of student interventions, the majority consisted of *teacher-led responses* (58,62%), arising from answers to the teacher. Other interventions included *explanations* (24,14%) and *student initiatives* (6,90%). *Explanations* were primarily used to clarify the methodology each group employed to solve the problem, while *student initiatives* reflected the students' engagement and interest in both the activity and the problem itself. Despite the high frequency of teacher actions, which aligns with Drageset's (2014) observations of teacher control in mathematical discussions, this may potentially limit student-led dialogue. In our study, more students tended to participate in response to closed or highly guided questions, whereas open-ended questions resulted in fewer but more elaborated explanations, allowing students to explain the strategies they used and the rationale behind their results. This

pattern may reflect the nature of the discussion, which focused primarily on concluding the activity, highlighting key points, and drawing conclusions after students had already discussed the problem with their peers.

Regarding objective 2, several patterns were identified during the class discussion. Firstly, the interactions observed followed the IRE (initiation-response-evaluation) pattern (Cazden, 2001). This finding is consistent with those reported by Drageset (2015b). In general, this approach constrains students, as the teacher is directing the discussion and there is limited opportunity for spontaneous participation. In consequence, the teacher's control during the class discussion indicates that student participation was reactive rather than initiated by the students themselves. It is important to note that, due to time constraints and the format of the whole-class discussion, not all students had the opportunity to speak. Typically, one member of each group presented the work of their group, summarizing collective contributions. While this approach allowed the discussion to proceed efficiently, it may limit the teacher's ability to validate the reasoning of every individual student. However, this predominance of the IRE pattern and the high proportion of teacher talk should not be interpreted solely as a traditional or restrictive teaching style. Instead, in the context of complex modeling tasks like Fermi problems, this structure functions as a form of pedagogical scaffolding. The teacher acts as an orchestrator who uses the "Evaluation" phase not to close the dialogue, but to redirect and deepen student reasoning (Wells, 1993).

In terms of the nature of the interactions observed, three distinct patterns emerged. The first was *Open progress initiative (Teacher) – Explanation (Student)*. This pattern reflects instances in which students were prompted to explain their reasoning and solution process. This fact is also observed in the second pattern *Explanation (Student) – Recap (Teacher)*, where following the student's explanation, the teacher typically summarizes and rephrases the information presented to ensure its comprehensibility for all parties. The third pattern observed is *Closed progress details (Teacher) – Teacher-led responses (Student) – Recap (Teacher)*. This emphasizes the pivotal role of the teacher during the discussion, as their actions appeared to play an important role in advancing the discussion to more complex aspects of the problem. These patterns are similar to those described by Drageset (2015b), who also emphasizes how teacher-directed interactions guide the flow of the discussion. In Drageset's study, the teacher's actions (such as focusing on specific details and advancing the discussion forward) often prompted structured responses from the students, like the *teacher-led responses* seen in this study. The teacher's role in guiding the conversation, particularly through the clarification of explanations and the navigation of problem-solving steps, aligns with Drageset's (2015b) observations regarding the way teachers facilitate and direct classroom discourse. It should be noted that the findings presented are based on the analysis of a limited number of student contributions during a whole-class discussion, which constrains the scope of the interpretations.

## Conclusions

In this study, we have analyzed the class discussion dynamics after solving a Fermi problem in groups. Our focus was on teacher actions and student interventions and interaction patterns. It has been observed that the teacher plays a significant role in guiding the discussions, utilizing focusing (*recap*, *notice* and *justification*) and progressing actions (*closed progress details*, *open progress initiatives* and *demonstration*), which aligns with Drageset's (2014) findings. These actions appeared to play an important role in clarifying concepts and solving methods, as well as for connecting different student ideas and potentially providing opportunities to refine and extend their mathematical understanding. Despite the teacher speaking the most, students contributed significantly by offering explanations, which allowed the teacher to highlight key points and advance the discussion toward more complex aspects of the problem.

The primary structure of the discussion was based on the IRE (initiation-response-evaluation) pattern (Cazden, 2001). Furthermore, three additional patterns have been identified: *open progress initiative* (teacher) – *explanation* (student); *explanation* (student) – *recap* (teacher);

and *closed progress details* (teacher) – *teacher-led responses* (student) – *recap* (teacher). The findings permit the identification of predictable scenarios that may arise during a class discussion, two of which were also identified in Drageset's (2015b) study. With students' input, the teacher guided the discussion constructively, supporting collective understanding and promoting engagement. Key aspects related to the viability and robustness of the solution processes (such as the space occupied by different people) encouraged students to reflect on and refine their strategies. Promoting such situations in the classroom may foster more complex reasoning and critical thinking. Overall, these findings highlight concrete ways in which teachers can orchestrate discussions to deepen students' mathematical reasoning, offering actionable guidance for classroom practice.

This study contributes to the field of mathematics education by examining how mathematical modeling and classroom discussions unfold after solving a Fermi problem in groups. The findings suggest that teacher-guided discussions can influence students' mathematical reasoning and problem-solving abilities. The two principal models employed (*reduction and use of proportion*, and *grid distribution*) illustrate that primary students are capable of applying sophisticated reasoning to real-world problems (Albarracín & Gorgorió, 2014; Peter-Koop, 2009). However, they still benefit from teacher intervention, as they are not yet fully autonomous. The class discussion enabled students to evaluate the effectiveness of their models and consider alternatives. Well-structured teacher-guided discussions can scaffold reasoning, support engagement, and foster collective understanding, particularly when teachers strategically combine different actions to highlight key points, connect ideas, and prompt reflection. Overall, the study highlights the potential of classroom discussions after solving a Fermi problem to validate student contributions, refine strategies, and promote critical and flexible thinking. By identifying predictable discussion scenarios, the study offers a tool to support teacher training and help educators make informed decisions during whole-class discussions of the solution processes for a Fermi problem.

### *Limitations of the Study*

The limitations of the study are its small sample size and the focus on a single Fermi problem situated in a familiar context. While this allowed us to complete a detailed analysis of the situation, it also restricts the generalization of the findings. Further research with a larger sample or different Fermi problems would be beneficial for a more comprehensive understanding of the phenomena. Additionally, the analysis focuses on observable discourse patterns rather than on the participation of all students, as not all students contributed orally during the discussion. As a result, the identified patterns reflect the interactions of a subset of the classroom and should be interpreted with caution. Furthermore, the reliance on Drageset's framework focuses the analysis primarily on teacher-student verbal interactions. While this provides a systematic lens to examine instructional moves, it necessarily delimits the exploration of other dimensions, such as the semiotic role of gestures or the long-term evolution of individual mathematical conceptions that other frameworks might emphasize.

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