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Thinking through Fermi problems: What proximity of the problem context is more attainable to promote students' model-based estimations?

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ABSTRACT

This study presents an empirical analysis of 6th-grade students' estimation-based thinking in two Fermi problems that are with different proximity to the problem context based on the following characteristics: (1) involving real-life elements within/beyond-direct-reach of students' physical surroundings and (2) involving the size of the quantities within/beyond-direct-reach of students' reference tools for estimation. Twelve turkish 6th-grade students worked in pairs on the Fermi problems. The findings indicated that the proximity of the problem context shaped students' model-based estimation on Fermi problems by affording processes characterized as sensemaking-facilitator, estimation–referent identifier, and strategy-promoter. Students' model-based estimation indicated that the closer proximity of the context helped students to make sense of the problem and initiate an estimation relying on within-direct-reach reference tools. However, the lack of physical closeness and the larger size of the quantity created a challenge in developing a model-based estimation, which encouraged students to develop a strategy.

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Model-based estimation; Fermi problems; proximity of the problem context; affordances of context proximity

Introduction

Exploring students' ways of thinking in problem solving caught the attention of mathematics educators, particularly in mathematical modeling. Although mathematical modeling has awakened the interest of the educational community due to its potential to describe complex phenomena and promote critical thinking, its inclusion in the classroom is still scarce (Borromeo Ferri et al., 2021). Given this situation, it seems necessary to deepen the investigation of activities that promote students' divergent ways of thinking in the modeling process.

One proposal is to use the so-called Fermi problems – open-ended estimation-based problems (Ärlebäck, 2009) that allow the establishment of classroom dynamics in which students experiment with the mathematical modeling processes (Haberzettl et al., 2018) and construct mathematical ways of thinking by themselves (Albarracín & Gorgorió, 2014). An example of a Fermi Problem could be "How many teachers are in your city/country?" Fermi problems are not only useful for students to develop contextualized estimates in mathematics (Ärlebäck, 2009) but also have been used in a variety of STEM education domains (Holubova, 2017; Morgan, 2017; Robinson, 2008). In this sense, Fermi problems go beyond intuitive estimation by engaging students in a process called model-based estimation (Ärlebäck et al., 2024). We use the term model-based estimation to refer to an approach to approximation in which a numerical estimate is generated through the explicit construction and use

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of a mathematical model, rather than intuition alone. In contrast to computational or measurement estimation (Hogan & Brezinski, 2003), where a figure is proposed without a structured rationale, model-based estimation involves a strategy of decomposing the problem into subproblems, making, and justifying assumptions, and establishing quantitative relationships that connect available data, external sources, or reasonable suppositions (Albarracín & Ärlebäck, 2019). It thereby aligns closely with mathematical modeling processes, as the estimation becomes the outcome of a reasoned and transparent chain of argumentation. In this example, the range of estimation could be a familiar context (like the city) or could be widened to a less accessible context (like the entire country). This adjustment of the problem context is pedagogically approached as "upscaling" in a Fermi Problem sequence (Albarracín et al., 2022). In this sense, upscaling the context of a Fermi problem means changing the context proximity addressed in the problem (Albarracín & Ärlebäck, 2025) and promoting estimation less relying on personal knowledge but requiring a more abstracted way of thinking – model-based estimation – due to the physical inaccessibility of the upscaled context (Jones et al., 2009).

Fermi problems ask for estimating different types of quantities, for which different model-based estimation strategies can be employed. For example, estimating objects in a line is different from estimating objects in a surface, and both are different from estimating quantities associated to social behaviors, like estimating the number of pictures uploaded to Instagram in a day (Albarracín & Gorgorió, 2015; Pla-Castells & Ferrando, 2019). Hence, the cognitive processes involved in the model-based estimation may differ depending on the elements involved in the Fermi problems. In this regard, previous research shows that Fermi problems with a similar mathematical formulation but located in different contexts promote different ways of thinking (Ferrando et al., 2021). This fact marks the beginning of a line of research that explores the potential sequences of Fermi problems in which the variations are introduced in the context, such as the pedagogical approach of upscaling (Albarracín et al., 2022). Considering students' struggle in navigating between contextual understanding and abstract mathematical reasoning in modeling real-world phenomena, particularly when faced with unfamiliar or complex contexts (Jupri & Drijvers, 2016), the upscaling approach provides a structured pathway through which learners can build and refine their model-based estimation strategies incrementally, moving from familiar to unfamiliar situations.

However, up to now, there has been limited research involving a variety of problem sequences, and we have little knowledge regarding the design criteria for problem context sequences. Given this gap, there is a need to understand in greater detail how the proximity of the problem context might influence students' ways of thinking. In response to this need, we aimed to investigate 6th-grade students' thinking processes in two Fermi problems, the context of which differed in proximity to students' lives. Specifically, the problems involved (1) elements that are within-direct-reach (vs. beyond-direct-reach) of students' physical surroundings (e.g., their own school vs. the entire country), and (2) size of the quantities that are within- direct-reach (vs. beyond- direct-reach) of students' reference tools for estimation (e.g., directly referencing their own height or their classroom size vs. developing estimation strategies). Aligned with this study design, we aim to explore the following research questions (RQ):

> RQ 1: What affordances of context proximity in Fermi problems emerge as students develop model-based estimation in Fermi problems involving (a) real-life elements within-direct-reach (vs. beyond-direct-reach) of students' physical surroundings and (b) quantity sizes within-direct-reach (vs. beyond-direct-reach) of students' reference tools?
>
> RQ 2: Which proximity (i.e., close versus distant) of the Fermi problem context is more attainable for promoting students' model-based estimation?

Such a design-oriented exploration of context proximity will inform us not only about the design of richer problem sequences but also about teacher training, particularly regarding the identification of contextual features that can be tailored to ensure problems are attainable for students and support the development of efficient estimation-based approaches. Given the potential of Fermi problems as "simpler but effective way to engage in interdisciplinary thinking, modeling, and model-based

estimation, and thus STEM thinking" (Ärlebäck et al., 2024, p. 4), identifying the potential affordances of the proximity of the context and sequencing problems in the trajectory of STEM education constitutes a fruitful research path.

Literature review

As it captures the cognitive processes that students engage in to analyze real-life situations and generate novel ways of thinking, mathematical modeling provides us with a relevant theoretical framework. In addition, we examine the use of Fermi problems in mathematics education as a medium for mathematical modeling.

Mathematical modeling for the development of divergent ways of thinking

Mathematical modeling is a problem-solving process where problem-solvers analyze a complex real-life situation, mathematize the situation with the help of internal conceptual structures, express through representation tools, such as spoken or written language, graphs, concrete models, and metaphors, and produce a mathematically efficient and contextually appropriate ways of thinking (Lesh & Doerr, 2003; Lesh & Sriraman, 2005). Grounding in Realistic Mathematics Education (RME), contextual nature of the modeling problems has merits in developing divergent thinking styles (Güner & Erbay, 2021) and creative thinking skills (Sitorus & Masrayati, 2016) because they are not merely asking problem solvers to apply prescribed solution methods.

Although the mathematical modeling process is still a controversial topic among the community, the prevalent view presents it as a cyclic process between the real world and the mathematical world (Blum & Leiß, 2007; Cai et al., 2014; Galbraith & Stillman, 2006). The modeling cycle starts with understanding the problem and follows by simplifying, structuring, and mathematizing the situation. This mathematical work produces a way of thinking – a model – that needs to be tested and revised as many times as needed. Schoenfeld (1992) argued that this kind of problem-solving process requires more than mere calculation and deduction, rather it involves observation of the situation, identifying the regularities, patterns, or restrictions involved in the situation, developing conjectures that might be based on assumptions or estimations, and then evaluation of the conjectures. In this regard, mathematical modeling constitutes a promising avenue for fostering the development of engineering habits of mind (Karatas-Aydin & Isiksal-Bostan, 2023) and STEM-oriented thinking (Ärlebäck & Albarracín, 2024; Ärlebäck et al., 2024; Holubova, 2017).

A model highlights conceptual thinking processes and therefore entails understanding, describing, predicting, and mathematizing real-world situations by quantifying, algebraizing, and systematizing (Lesh et al., 2000; Mousoulides et al., 2007). The modeling process is creative (Wessels, 2017), in the sense that it involves fluency by considering various options to configure a model, flexibility by constantly choosing between different concepts or procedures to achieve the same end, and originality as unique solutions are created for a given problem. In this process, making assumptions and estimations plays an essential role in constructing a model because it demands students to take proactive actions based on their choices that would aid in developing an acceptable model (Anhalt & Cortez, 2015). In this sense, estimation-based and open-ended problems, such as Fermi problems, are potentially good candidates for initiating the thought-development process (Albarracín & Gorgorió, 2014; Ärlebäck, 2009) as they encapsulate the essential decision-making processes of mathematical modeling (Ärlebäck, 2011), enable creative thinking (Okamoto et al., 2023), and foster critical thinking (Sriraman & Knott, 2009). Thus, Fermi problems have been suggested as activities that promote mathematical modeling (Ärlebäck, 2009) and creative thinking (Lunt & Helps, 2001).

Fermi problems as a medium for developing model-based estimation

Ärlebäck (2009) describes Fermi problems as "open, non-standard problems requiring the students to make assumptions about the problem situation and estimate relevant quantities before engaging in, often simple calculations" (p. 331). Schukajlow et al. (2023, p. 419) categorized the openness of modeling problems into three categories: (1) Openness of the initial state, (2) Openness of the intermediate states, and (3) Openness of the goal state. According to this categorization, Fermi problems are open-ended in all three aspects. That is, such problems often do not have any numerical value in the problem statement, the solution process is not prescribed, and they do not require an exact answer as a solution. In this regard, Fermi problems are open to make realistic assumptions about the unknown quantities and to stimulate diverse ways of thinking. Furthermore, Fermi problems are characterized as a form of activity that requires the simplification and mathematization of a reality requiring estimations (Robinson, 2008) and promotes students in generating novel ways of thinking based (Henze & Fritzlar, 2010); therefore, act as modeling activities in the sense of Lesh et al. (2000).

Fermi problems ask students to make estimations for the quantities involved in real-life situations to construct models of those situations, rather than recalling and applying known facts and procedures (Segura & Ferrando, 2023). Hogan and Brezinski (2003) distinguished three types of estimation: numerosity, measurement, and computational estimation. While numerosity estimation involves estimating the number of items distributed in an array, measurement estimation concerns estimating a property of an object (height, weight, length, etc.). Computational estimation differs from these two in the sense that it involves estimating the result of one or more arithmetic operations. Nevertheless, to solve the Fermi problems, problem solvers tend to decompose the original problem into simpler sub-problems (Carlson, 1997) and employ strategies such as unit iteration and benchmark referencing (Weiher et al., 2022). Regarding the strategies for solving Fermi problems, Albarracín and Gorgorió (2014) proposed a characterization that includes (1) the use of external source, (2) exhaustive count, (3) reduction of the problem and use of proportion, (4) the use of concentration measures, (5) iteration of the measurement of a base unit of reference, and (6) the use of grid distribution. By using one or more of these strategies, problem solvers can produce reasonable estimates (Taggart et al., 2007), formulate educated guesses (Sriraman & Lesh, 2006), and engage in mathematical reasoning based on those estimates (Ridgway et al., 2001). Therefore, Fermi problems involve a different type of estimation, referred to as model-based estimation (Ärlebäck et al., 2024). This kind of estimation requires the development of a model of the real-life situation presented in the Fermi problem in order to make reliable estimates and may involve problem decomposition and/or the use of estimation strategies (Ärlebäck et al., 2024). Furthermore, model-based estimation targets real-world problems and is claimed to simulate STEM ways of thinking. In this regard, Ärlebäck et al. (2024) argued that Fermi problems can draw on students' collective and interdisciplinary knowledge and raise their awareness of the importance of contextual thinking in STEM projects.

We interpret that there are two types of possible contexts for the pedagogical use of the Fermi problems. The first type used situations where students can easily cope with artificial contexts through simple estimates. This would be the case of estimating the number of M&M's in a jar, which is a familiar but not relevant context to students and allows the introduction of mathematical concepts, such as the volume of an object. The second type used the contexts that can be real and allow obtaining new information to assess relevant situations. An example of this type is used in an astrobiology course to estimate the amount of energy used by humanity each year to compare it with the energy that the Earth receives from the Sun (Morgan, 2017). In research studies, it has been observed that students can be encouraged to develop strategic thinking through sequences of Fermi problems involving the same type of estimation across different real-world contexts in each problem (Albarracín et al., 2021).

A way of sequencing Fermi problems is the use of context variations – upscaling or downscaling the context. That is, Fermi problems are sequenced in progressively more distant or closer contexts within a similar problem setting (Pla-Castells & Ferrando, 2019). It has been confirmed that upscaling contexts, in the sense of starting from a close context and progressively solving problems in more

distant contexts, is an effective scaffolding strategy that encourages students to develop reliable estimations (Albarracín et al., 2022). The researchers argued that problem solvers may struggle to make sense of distant contexts without prior experience working with closed contexts. Following this line of research, in this study we conceptualized the proximity of the problem context in terms of two types of within-reach considerations (i.e., within-direct-reach versus beyond-direct-reach physical surroundings and within-direct-reach versus beyond-direct-reach reference tools for estimation) as a potential way of making problems *attainable* for students. We conjectured that problem contexts involving real-life elements within-direct-reach of students' physical surroundings and quantities whose sizes are within-direct-reach of students' reference tools for estimation may facilitate the development of estimation-based thinking in Fermi problems. This conjecture is based on both prior research on sequencing Fermi problems with respect to close versus distant contexts and our consideration that quantities extending beyond students' experiential worlds may be less attainable for them.

Methods

This exploratory study was designed to investigate 6th-grade students' estimation-based thinking process in Fermi problems developed considering the proximity of the problem context in two types: (1) within-direct-reach/beyond-direct-reach of students' physical surroundings, and (2) within-direct-reach/beyond-direct-reach of students' reference tools. Below, we explain the data collection and analysis procedures of the study.

Participants

This study[1] was carried out with 12 sixth-grade students (six girls and six boys), studying in public middle schools in six different cities in Türkiye. A short meeting with reachable mathematics teachers in different cities was held to explain the purpose of the research, and the teachers were asked to suggest some students as potential participants to work in pairs. Then 12 students were invited to participate in the study upon parental consent. Six groups (i.e., Group 1–6) were formed (one pair from each city), and they solved Fermi problems in an online session due to the Covid19 pandemic conditions. The students were invited in pairs due to two main reasons: (1) students who already knew each other might communicate better in an online platform, and (2) students who lived in the same city might make sense of their interpretations about the local situations more easily. Although we did not have access to students' academic records, they were suggested by their mathematics teachers based on their average or above average performances[2] in mathematics lessons and the skills of expressing their thoughts.

Data collection procedure

Due to the pandemic constraints, the data were gathered in an online platform, Zoom, and, the digital opportunities, such as the breakout rooms for students' group work, screen sharing to be able to work together, and annotating tools to edit on each other's work in pair-work time, were utilized to gather rich data from students. During the online problem-solving sessions, students in pairs were given a flexible time to develop solutions to two Fermi problems. The characteristics of the problems that identify the proximity of the problem context are explained in Table 1.

Although there is no clearcut borderline for the proximity of the problem context (i.e., within-direct-reach vs. beyond-direct-reach), these two problems indicated almost two opposite end points of the continuum of close-proximity versus distant-proximity context with respect to the real-life elements within/beyond-direct-reach of students' physical surroundings and the size of the quantity within/beyond-direct-reach of students' reference tools for estimation, as shown in Table 1. Furthermore, aligned with the nature of the Fermi problems, both problems were lack of numerical

Table 1. Fermi problems posed to the students.

		Proximity of the problem context	
	Fermi Problems	With the real-life elements within/ beyond-direct-reach of students' lives/physical surroundings	With the size of the quantity within/ beyond-direct-reach of students' estimation reference tools
Close proximity context	Student Weight Problem: What would be the total weight, in kg, of all students at your middle school?	Close to students' lives (i.e., asking at their school)	Close to students' reference tools (i.e., estimating based on their own or friends' weights)
Distant proximity context	Mall Count Problem: How many shopping malls can there be in Türkiye?	Far from students' lives (i.e., asking in the entire country)	Far from students' reference tools but could be adjustable by considering the number of malls in their own city

- What did you understand from the problem?
- What information do you need to solve the problem? Is the given information sufficient to solve the problem? Explain why/why not?
- Explain where you got the information you need for the solution but was not given in the problem?
- What are the solutions you produced?
- Could there be other information/situations affecting the solution of the problem? If yes; what
- information/situations might affect the solution of the problem? How can it affect?
- Do you think that you were able to reach an exact solution to the problem? Do you think that it is reachable?
- Do you think that you need an exact solution for the problem?
- How reasonable and/or approximate do you think that your estimate is as the solution of the Fermi problem?
- How would you improve the approximation of your estimate as the solution of the Fermi problem?

Figure 1. Semi-structured leading questions for the whole group discussion.

information. Specifically, the number of students in the school and the weight of a student were not specified in the Student Weight Problem, and students were expected to develop a model-based estimate for the total weight of the students at their school. Similarly, Mall Count Problem did not specify the number of shopping malls in a particular region for reference, for which students needed to develop a model-based estimation.

As students work on those Fermi problems, several breaks were provided in line with the students' request to ensure their well-being. After each group produced their solution in a separate room on the online platform, they were asked to share their solution with other groups and engage in a group discussion that the teacher-researcher led through the semi-structured list of questions (see Figure 1).

Students' problem-solving steps and their whole class discussions were video recorded. These video records, along with the transcriptions, were analyzed in depth to understand their ways of thinking when solving the Fermi problems and to identify the affordances that problem contexts entail at particular proximities for which the Fermi problems were designed.

Data analysis procedure

We employed a content analysis that involved deriving codes and categories from the students' written solutions and verbal explanations (Miles et al., 2014). When the codes were associated to the solution steps, the categories were determined by identifying the structure of the solution indicating their model-based estimations (see code table in Table 2). To compile the code table, we started with deductive approach and listed the solution strategies based on the framework proposed by Albarracín and Gorgorió (2014), which characterized student solutions to Fermi problems. However, not all the strategies proposed by experts – such as exhaustive count and grid distribution – were applicable for

Table 2. Code table used to analyze students' solutions.

Solution Strategies	Model-based estimation strategies	Explanation
Iteration of a unit strategy	Based on a single estimate	Getting one estimation by personal knowledge, and then iterate it as many times as needed
	Creating a single estimate by taking the average	Considering several different estimates and calculating the average of them to get a more reasonable estimate
	Based on two single-estimates	Getting a rough estimation for each of the two quantities and then multiplying of the two single-estimates
Splitting the problem strategy	Splitting into two smaller problems	Splitting multiplicative situation into two smaller parts – *based on distributive thinking* – and make estimations for smaller parts
	Splitting into three smaller problems	Splitting multiplicative situation into three smaller parts – *based on distributive thinking* – and make estimations for smaller parts
	Splitting into four smaller problems	Splitting multiplicative situation into four smaller parts – *based on distributive thinking* – and make estimations for smaller parts
Scale-up strategy	Stratification to scale up	For the cases where the problem is split into smaller parts, using scaling up for each of the smaller parts and then adding each of the stratified scaling ups
	Proportionality to scale up	For cases involving a single estimate, using proportionality to scale up the estimate
	No scale up	Not using any multiplicative structure to scale up
Other strategies or no strategy	Algebraic formulation	Labeling the quantities that needs to be estimated as algebraic variables and developing an algebraic expression
	Making a random guess without any reference	Without using any reference for estimation, guessing a numerical result
	Not sense making	Not able to sense the problem situation and not able to develop an estimation-based thinking

the quantities involved in the Fermi problems used in this study; therefore, only applicable ones were adapted to the code list of the current study.

After identifying the solution category, we explored each solution type inductively to code model-based estimation strategies in detail. As we code deductively and inductively, we also wrote code memos to distinguish the nuances in ways of thinking. For instance, Group 3's solution to Mall Count Problem was "81 provinces in the country × 1000 markets in each province = 81000" simply indicated iteration of a unit estimate based on their personal knowledge and therefore coded as "iteration of a unit strategy" and "based on a single estimate." Furthermore, we wrote a memo about the source of this unit estimate of 1000 markets in each province, which was a TV advertisement they recalled and which led to or was the result of their confusion between malls and markets. Although this particular information was not marked as a code, it was noted as a code memo to capture the nuances in model-based estimations.

After analyzing each group's work on each problem, we carried out retrospective cross-comparison analysis based on the close versus distant proximity of the Fermi problem contexts. In particular, we looked for model-based estimations employed not only by different groups but also across different problems, which helped us to articulate the affordances of the problem context at different proximities for students. Table 3 presents the matrix used in the cross-comparison analysis, noting that the categories in the rows were grounded in the data analysis held by using the code list in Table 2 while the proximity categorization derived from the study design. The group number of students and their model-based estimation strategies were placed in the empty cells during the comparative analysis.

Similar to the first stage of coding, while this cross-comparison matrix is built, both students' written calculations and their written and oral explanations were considered. For instance, Group 3's abovementioned coding and the code memo in the Mall Count Problem indicated "estimation-referent identifier" as an affordance of the context proximity because the distant proximity of the context led them to find an estimation-referent, for which they used the TV advertisement. This solution did not support the "sensemaking-facilitator" affordance because TV advertisement, used as a reference, either caused or was caused by a lack of correct sensemaking of the context (i.e., considering grocery markets instead of malls). Hence, the affordances of context proximity were

Table 3. Cross-comparison analysis matrix.

AFFORDANCES OF THE CONTEXT PROXIMITY	PROXIMITY OF THE PROBLEM CONTEXT **Close-Proximity: Within-direct-reach of students' lives and reference tools** *Student Weight Problem*	**Distant-Proximity: Beyond-direct-reach of students' lives and reference tools** *Mall Count Problem*
Sensemaking-facilitator – Context proximity is used to make sense of the problem – Context proximity is used to make sense of the estimating quantity	*This part of the comparison matrix is filled by the group # and the group's model-based estimation strategies promoted by the affordances of the context proximity.*	
Estimation-referent identifier – Context proximity provided a reference tool – Context proximity provided an estimation unit to iterate		
Strategy-promoter – Context proximity encouraged splitting strategy to develop more reliable estimate – Context proximity encouraged getting a smaller estimate to scale up		

derived inductively from students' written work and small-group discourse and clustered into three stable analytic categories, which we report as the affordances of context proximity: (i) sensemaking facilitator, when proximity supported interpreting the situation and setting reasonable assumptions; (ii) estimation-referent identifier, when proximity enabled plausible anchor quantities or units; and (iii) strategy promoter, when proximity made particular modeling moves (e.g., splitting or scaling) salient and workable.

To sum up, we first coded each group's model-based estimation structure (e.g., unit iteration, splitting, scaling, or algebraic formulation) and then coded how students leveraged context at key decision points (sensemaking, reference-quantity selection/justification, and strategy choice). Constant comparison across groups and proximity conditions yielded three stable categories of affordances: sensemaking-facilitator, estimation-referent identifier, and strategy-promoter. Furthermore, the data analysis was triangulated by a second coder, and all unmatched codes between the analysis of two-coders were discussed until full agreement was reached by both coders (Lincoln & Guba, 1985). In addition, all the codes and categories were discussed by all three researchers. Furthermore, cross-comparison analysis allowed us to compare different unit of analysis (solutions within the problem across groups of students vs. solutions across the problems), which contributes to the credibility of the analysis (Miles et al., 2014).

Findings

In this section, we present the affordances of the problem context that supported students' model-based estimations in their solutions to two Fermi problems (addressing the RQ1) and then the cross-comparison of these two cases of close- versus distant-proximity of the problem context (addressing the RQ2).

Sensemaking-facilitator affordance of the context proximity

Because Fermi problems provide no numerical information, students had to engage in sensemaking by clarifying what the target quantity means and committing to reasonable assumptions before they could

Table 4. Students' model-based estimations based on sensemaking facilitated by the context proximity.

Problem	Group	Solution	Model-based estimation
Student Weight Problem	Group 2	40 kg and 50 kg are predictions of group members. Average of 40 kg and 50 kg is 45 kg 18 = Number of classes 25 = Number of students in each class 18 × 25 = 450 students in total. 450 × 45 kg = 20250 kg in total.	- Iteration of a unit strategy - Based on a single estimate - Creating a single estimate by taking the average - Proportionality to scale up
Mall Count Problem	Group 1	"There should be 550 shopping malls in Türkiye."	- Making a random guess without any reference - No scale up
	Group 4	100 = estimated number of shopping malls per province 92 = number of provinces 100 × 92 = 9200 shopping malls in Türkiye	- Iteration of a unit strategy - Based on two single-estimates - Proportionality to scale up
	Group 5	x is the number of shopping malls in Muğla (i.e., relatively smaller city) and y is the number of shopping malls in İstanbul (i.e., a metropolitan city). So, the total number of malls in Turkiye should be the average of $\frac{x+z}{2}$ *where z is the range between ×and y*	- Algebraic formulation - No scale up

produce any model-based estimate. Accordingly, we define the sensemaking facilitator affordance not as context proximity *forcing* sensemaking, but as context proximity *enabling* sensemaking toward the development of more feasible estimations by drawing on experiential constraints and familiar interpretive resources. Therefore, we observed this affordance in all groups, except Group 1, in both close and distant proximities of the context. On both problems, Group 1 could not establish any strategy for developing a solution although they directly made a rough prediction without further explanation on the second problem, Mall Count Problem. Other groups' solutions indicated that the proximity of the problem context enabled them to make sense of the situation and to develop a model-based estimation.

In Table 4, we illustrate only those groups whose solutions primarily exemplify the sensemaking facilitator affordances of context proximity, although this affordance was also observed in other groups' solutions in combination with additional affordances.

As seen in Table 4, on the Student Weight Problem, Group 2 decided to make an estimation for the weight of a so-called random middle school student, and to do so one student in Group 2 suggested 40 kg as an estimate and the other suggested 50 kg. The close-proximity context facilitated students' making sense of the situation based on their personal knowledge, and then they took the average of these two personal suggestions and identified the estimated weight for any student in the middle school to be 45 kg. Afterward, they made two more estimations: one is the estimation for the number of classes in the school, regardless of the grade level, and the other one is the number of students in each class. The final multiplicative structure indicated the proportionality to scale up the weight of a random student to the weight of all students in the school.

On the Mall Count Problem, Group 5 produced an algebraic solution rather than a numerical estimate. Their way of thinking was based on the idea that the number of malls would be different depending on the size of the cities, similar to Group 2 in that sense. So, they separately labeled the number of malls in a smaller city where they live, and the number of malls in the most populous city, Istanbul. They thought that the range between those two numbers needed to be used to find the best estimate. They decided to take "the average of the range and the estimate for the smaller city," in their own words, however, they did not explain what this average mean and why it would make sense to find the estimation of the total number of malls in the country. It was also not clear whether this average was valid for only small cities and so needed to be multiplied by the number of small cities. Although this group could not produce a reasonable estimation, students attempted to make the problem context at a distant-proximity more attainable, indicating the sensemaking-facilitator affordance of the context proximity.

Estimation-referent identifier affordance of the context proximity

We observed in some solutions that the proximity of the problem context provided a reference tool such as students' own weight on the Student Weight Problem and a TV advertisement on the Mall Count Problem. In such cases, the context proximity functioned as an estimation-referent identifier based on which students developed their model-based estimations. As seen in Table 5, the context proximity offered an estimation reference for four of the six groups on the Student Weight Problem and for two groups on the Mall Count Problem. This indicated that the close proximity of the problem context offered students more opportunities to identify some references (e.g., their own class population, their own weight, their siblings' weights).

On the Student Weight Problem, Group 3, 4 and 5 estimated the weight of students at each grade level separately, with increasing weight as the grade level increases. This approach indicated that the students found the problem context within their attainable proximity. One of the students in Group 3 stated: "The weight of the students in the lower and upper-grade levels must be different. Is your weight (*referring to his partner*) the same with your elder brother in 8th grade?" Hence, the context that is within-direct-reach of students' lives enabled them to make realistic observations, which resulted in

Table 5. Students' model-based estimations based on references identified by the context proximity.

Problem	Group	Solution	Model-based estimation
Student Weight Problem	Group 2	See Table 4.	See Table 4.
	Group 3	30 = Number of students in each class 6 = Number of classes in each grade level. 30 × 6 = 180 the total number of students at each grade level 40 kg = average weight estimate for 5th graders 180 × 40 = 7200 kg for 5th graders 50 kg = average weight estimate for 6th graders 180 × 50 = 9000 kg for 6th graders 55 kg = average weight estimate for 7th graders 180 × 55 = 9900 kg for 7th graders 60 kg = average weight estimate for 8th graders 180 × 60 = 10800 kg for 8th graders 7200 + 9000 + 9990 + 10800 = 36900 kg is the total weight	- Splitting the problem strategy - Splitting into four smaller problems - Stratification to scale up
	Group 4	Weight estimation for 5th, 6th, 7th and 8th graders, respectively 48 kg, 58 kg, 68 kg, and 58 kg. *Total weight for students at each grade*: 12 × 48 = 576 10 × 58 = 580 16 × 68 = 1088 18 × 58 = 1044 576 + 580 + 1088 + 1044 = 3288 kg total weight	- Splitting the problem strategy - Splitting into four smaller problems - Stratification to scale up
	Group 5	Estimation of the total number of students: 264 + 264 + 297 + 297 = 1122 *Estimation of the total weight* The sum of their own and their friends' weights: 40 + 45 + 55 + 60 = 200 Average weight for a student: 200 ÷ 4 = 50 kg 11122 × 50 = 56100 kg total weight	- Splitting the problem strategy - Splitting into four smaller problems - Iteration of a unit strategy - Based on single estimate - Creating a single estimate by taking the average - Proportionality to scale up
Mall Count Problem	Group 3	81 = the number of provinces in Türkiye 1000 = the number of markets in each province 81 × 1000 = 81000 shopping malls in Türkiye	- Iteration of a unit strategy - Based on single estimate - Proportionality to scale up
	Group 6	*For small-sized cities*: 55 × 1 = 55 shopping malls *For large-sized cities*: 3 × 15 = 45 shopping malls *For medium-sized cities*: 23 × 6 = 138 shopping malls 55 + 45 + 138 = 238 shopping mall in Türkiye	- Splitting the problem strategy - Splitting into three smaller problems - Stratification to scale up

not having one grand estimate indicating the weight of a middle school student. Similarly, Group 4 and 5 identified different estimates for each grade level based on the references relying on their real-life observations. Particularly, Group 5's estimate for the average weight was based on their own and friends' weights at the lower and upper grades, indicating that they considered self-related references for making the estimation of the quantity. This observation showed that the students conceived the size of the quantity within-direct-reach of a reference tool for the estimation, which indicated the estimation-referent affordance of the problem context. Furthermore, as these three groups had different estimates for different grade levels, their solution approach led them to do stratification, which is the strategy that is explained in the following section.

On the Mall Count Problem, the distant-proximity of the problem context inhibited students identifying some references to make legitimate estimates. Nevertheless, Group 3 recalled a TV advertisement stating, "1000 branches of A101 [*the name of the grocery store in the advertisement*] all over Türkiye." Therefore, they set their estimation as 1000 grocery stores in each province and proportionally scaled up their estimation. For this group, the distant-proximity of the problem context activated the need for an estimation-referent identifier, so they used the TV advertisement to make their estimation. On the other hand, it is important to note that Group 3's numerical estimations was distinctly different from other groups' solutions because they misperceived and misinterpreted the shopping malls as grocery stores (markets, in other words), and in fact this misinterpretation made them use the advertisement content as the estimation-referent identifier. Group 6, on the other hand considered the number of shopping malls in their city as a reference while estimating the mall counts for different-sized cities. Therefore, the proximity of the problem context stimulated the estimation-referent identifier affordance for this group. As also seen in this group's solution, this affordance is accompanied to the strategy-promoter affordance of the context proximity, which we explain in the next section.

Strategy-promoter affordance of the context proximity

Although we expect students develop model-based estimations to solve both FPs, it was not always the case as we saw in Group 1. Although Group 3 and 4 performed better and at least showed that they made sense with the context, they could get a single estimate and iteration of that unit estimate on the Mall Count Problem (see Tables 4 and 5). As mentioned earlier, the Fermi problems suggest a model-based estimation which often involve a splitting strategy – splitting the problem into smaller attainable pieces. We observed that the context proximity of problem promoted such a strategy for four of the six groups on the Student Weight Problem and for two groups on the Mall Count Problem (see Table 6).

On the Student Weight Problem, Group 2, 3, 4, and 6 split the problem into four smaller parts; that is, they estimated the weight of students at each grade level separately, the number of students in each class and/or the number of classes at each grade level. This splitting strategy enabled them getting more reliable estimates for the solution. Yet, there were some nuances in the overall solutions. For instance, Group 4's solution was similar to Group 3 in the sense that they made different estimates for different grade levels, but different regarding the estimation of the school size. Group 4 did not think that there could be more than one class at each grade level; rather they got quite low estimate for the number of students at each grade level. Still, their solution also indicated stratified choices of estimates to scale up, which the close-proximity of the problem context enabled students to develop (i.e., strategy-promoter affordances). Another difference between Group 3 and Group 4 was the estimates assigned to the weights of students at each grade level. As Group 3's estimates increased by grade level, Group 4's estimate did not exactly follow that pattern (the weight estimates were 48 kg for 5th graders, 58 kg for 6th graders, 68 kg for 7th graders but 58 kg for 8th graders), which we do not have any evidence to explain why. Group 6 also split up the problem into four as if there were separate problems for each grade (i.e., strategy-promoter affordance). In contrast, they used a fixed number of students at each grade level instead of having different estimates for each grade. Therefore, Group 6's model-based estimation indicated neither a sum of partial products nor a simple multiplication structure; rather it

Table 6. Students' model-based estimations based on strategies promoted by the context proximity.

Problem	Group	Solution	Model-based estimation
Student Weight Problem	Group 2	See Table 4.	See Table 4.
	Group 3	See Table 5.	See Table 5.
	Group 4	See Table 5.	See Table 5.
	Group 6	Weight estimation for 5th, 6th, 7th and 8th graders, respectively 38 kg, 45 kg, 50 kg, and 55 kg. Total weight for students in each grade level: $180 \times (38 + 45 + 50 + 55) = 6840 + 8100 + 9000 + 9900 = 33840$ kg total weight	- Splitting the problem strategy - Splitting into four smaller problems - Stratification to scale up
Mall Count Problem	Group 2	An estimate for the number of malls as an average can be 20 in big cities and 5 in small cities. There are 30 big and 51 small cities in Türkiye. $20 \times 30 = 600$ malls in big cities $5 \times 51 = 255$ malls in small cities There are 855 in total.	- Splitting the problem strategy - Splitting into two smaller problems - Stratification to scale up
	Group 6	See Table 5.	See Table 5

was represented a distributive reasoning of the fixed number of students at each grade level over the sum of weight average estimates for each grade level.

On the Mall Count Problem, Group 6 aimed at finding the number of malls in small, medium, and large cities separately and so split the problem into smaller problems. More specifically, this group considered the following aspects of the problem situation: (1) the number of malls is not equal in the east-west and north–south parts of the country due to the differences in geographic conditions and population, and (2) the size of the cities may influence the number of malls. They stated: "There seems to be fewer malls in the eastern part of the country than in the west, and logically there must be fewer malls in small-sized cities than big cities." Therefore, they made different estimations for the number of different-sized cities in the country, and the number of malls in different-sized cities.

Overview of students' model-based estimation in relation to the affordances of context proximity

The overview of the findings addresses the second research question: which proximity (i.e., close versus distant) of the Fermi problem context is more attainable for students, given the emerging affordances of context proximity. Students' abovementioned work showed that they could develop more detailed solutions to the Student Weight Problem, indicating that the Fermi problem at a closer proximity was more attainable for students. Although most of the groups could develop a solution to the problems, the strategic approach they held indicated some differences in the model-based estimations. While some of them indicated more rough estimations (e.g., Group 2), some involved stratification (separate estimates) and led to more detailed ways of thinking (e.g., Group 3, 4, and 6). While some involved estimations based on their own or friends' circumstances (e.g., Group 3 and 5), some were based on their observations of students at upper and lower grades (e.g., Group 3, 4, and 6). Hence, we could observe that the elements of problem context including the size of the quantity and closeness of the context to students' lives functioned as facilitating the development of model-based estimations of all groups, except Group 1 who did not produce any solution. Specifically, the sensemaking-facilitator affordance of the problem context mediated comprehension of the situation, estimation-referent identifier affordance encouraged students to identify a referent for estimations since the size of the quantity and the closeness of the context allowed students to make a reference-based estimation (e.g., Group 4 and 6), and strategy-promoter affordance stimulated solution strategies such as splitting the problem into smaller parts.

The Mall Count problem, on the other hand, presented a distant-proximity context which was beyond-direct-reach of students' lives and reference tools. However, the problem context's not being close to students' lives and the size of the quantity's not being within the range of their direct estimation led students to develop strategies, such as splitting the problem into smaller ones and identifying relevant issues that might provide references to make more reasonable estimation (i.e., considering geographic conditions in the east and west of the country and population differences in different-sized cities that would potentially affect the number of malls). Therefore, a particular proximity of the problem context functioned as a strategy promoter for developing more sophisticated ways of thinking in model-based estimation situations. Table 7 presents a synthesized cross-comparison analysis of students' model-based estimations in relation to the affordances of the context proximity.

Table 7 shows that almost all groups engaged in the Student Weight Problem in which the close proximity context helped them to make sense of the problem, identify an estimation reference, and develop a strategic approach for model-based estimation. On the other hand, distant proximity context functioned differently for different groups; that is as a sensemaking-facilitator for the groups 1, 4, and 5, as an estimation-referent identifier for the groups 3 and 6, and as a strategy-promoter for the groups 2 and 6. This indicated that especially the groups 4 and 5 found the Mall Count Problem difficult to make sense with the estimations although they could identify a reference tool for a reliable estimation and even Group 4 used splitting strategy to construct a model-based estimation. Students' work on the first problem signaled that they could develop a model-based estimation when and if they found the problem at the attainable proximity. Furthermore, when the problem context is not at the range of their attainable proximity, the context proximity served mainly as sensemaking-facilitator. Although we could compare only two Fermi problems here and our analysis did not indicate a causal relationship, the pattern of each group's work in these two different proximity cases enabled the comparison of the affordances for the attainable context proximity of the Fermi Problems.

In Fermi problems where it is necessary to provide an estimate, if students had more knowledge about the context, they could provide an estimation based on reliable information. Therefore, the closer proximity of the problem context – both the physical closeness of the context and narrower the size of the quantity on the Student Weight Problem generated – sensemaking-facilitator, estimation-referent identifier, and strategy-promoter affordances, which helped students' estimation-based thinking. In addition, there were occasions of students struggling with the distant-proximity on the Mall

Table 7. Affordances of the context proximity in students' model-based estimations.

	Model-based estimations in different context proximities			
Affordances of the context proximity of fermi problems	Close-proximity: within-direct-reach of students' lives and reference tools *Student Weight Problem*		Distant-proximity: beyond-direct-reach of students' lives and reference tools *Mall Count Problem*	
Sensemaking-facilitator*	Based on a single estimate	• Group 2	Making a random guess without any reference Based on two single-estimates Algebraic formulation	• Group 1 • Group 4 • Group 5
Estimation-referent identifier	Creating a single estimate by taking the average Iteration of a unit strategy	• Group 2 • Group 3 • Group 4 • Group 5	Based on a single estimate Iteration of a unit strategy	• Group 3 • Group 6
Strategy-promoter	Splitting the problem strategy (splitting into 4 problems) Scale-up strategy (proportional and stratified)	• Group 2 • Group 3 • Group 4 • Group 6	Splitting the problem strategy (splitting into 2 or 3 problems) Scale-up strategy	• Group 2 • Group 6

*For the groups listed in this row, the context proximity functioned dominantly a sensemaking-facilitator, yet the groups listed in other two affordances were also supported by the sensemaking-facilitator affordance.

Count Problem – involving wider size of the quantity. Students' difficulty in scaling the estimation from close to far context encouraged splitting the problem into smaller ones and produced a strategy-promoter affordance. Depending on the problem structure, splitting strategy demonstrated some variations such as splitting two, three, or four problems, which produced differences in the estimation values while preserving the same model-based estimation structure.

Discussion and conclusion

In this study, we presented the work of 12 students who worked in pairs on two Fermi problems, Student Weight Problem and Mall Count Problem. These two problems were different regarding two main characteristics of the proximity of the problem context: (1) involving the elements that are within/beyond-direct-reach surroundings of the problem-solvers (i.e., own school, own city), and (2) involving the quantities in the size that is within/beyond-direct-reach of problem-solvers' references for estimation (e.g., referencing to own weight, classroom size, mall counts in own city). The Student Weight Problem was more proximal to the students' own lives compared to the Mall Count Problem. Specifically, the latter addressed a wider range quantity (i.e., the entire country) that was not easily attainable by students within their physical circumstances. On the other hand, Student Weight problem asked for the estimation "at your school" and so involved a narrower, more specific, and mentally and physically attainable size of the quantity. Below, we discuss the findings addressing the affordances of the problem context in different proximities (i.e., close-proximity versus distant-proximity) in 6th grade students' model-based estimations.

What happens if the problem context is not at an attainable proximity for students?

Based on the cross-comparison analysis of students' work within and across the Fermi problems, we observed that Group 1, the lowest-performing group among the six, could not make sense of the open-ended nature of the problems. Therefore, Group 1 could not produce a model-based estimation for the Student Weight Problem. They made a rough random estimate for the number of malls in the country, indicating that the problems were not in their attainable zone of proximity either. A possible explanation for their random estimate in the second problem could be that they observed other groups shared their work on the first problem. Thus, Group 1 might have realized that such problems require estimation, and so they could make a random guess in the second problem. Since they stated a couple of times that they did not know what to do, we think that the problem context was not attainable to those students, given that they were not familiar to such problems. Considering that Fermi problems are open-ended in terms of initial, intermediate, and goal states when examined according to Schukajlow et al.'s (2023) classification, it is not surprising for students to have difficulty handling the ambiguity of Fermi problem contexts, especially those not involving numerical values. Since making sense of the problem context is the initial step in mathematical modeling (Blum & Leiß, 2007; Lesh & Doerr, 2003; Lesh & Sriraman, 2005), Group 1 could not pass this stage and could not find a way, such as splitting the problem as we observed in other groups, to move the problem to their attainable zone and develop a model-based estimation to the Fermi problems.

We note that the participating students were not selected based on their academic achievement in mathematics. Group May 1, or may not be relatively less successful than the other groups. In this regard, we do not have evidence to relate students' ways of thinking in Fermi problems to their mathematics achievement in this study, yet we can relate with the proximity of problem context to students' model-based estimations.

What happens if the problem context is at an attainable proximity for students?

The findings indicated that the proximity of the problem context shaped students' ways of thinking on the Fermi problems and produced three affordances: sensemaking-facilitator, estimation-referent

identifier, and strategy-promoter. Most students' solutions showed that both the physical closeness of the context and the narrower size of the quantity to be within-direct-reach of students' references helped them to make sense of the problem and develop a model-based estimation. Yet, the problem context in distant-proximity created a challenge for students and encouraged them to develop a strategy to manage this challenge (i.e., estimation-referent identifier and strategy-promoter affordances), which we discuss in detail below.

Close-proximity context facilitates, distant-proximity context invokes model-based estimation

On the Mall Count Problem, Group 2 splits the problem into smaller parts that could be made sense easily (i.e., estimating the number of malls in big cities and small cities separately). On the other hand, on the Student Weight Problem, they made a single estimate for the weight of a middle school student (i.e., iteration of a unit strategy, Ferrando & Albarracín, 2021), not considering the weights of 5th, 6th, 7th, and 8th graders differently. The closeness of the context to their own physical circumstances led them to produce an estimate quickly by referencing their own weight, not necessarily the most plausible estimate. In the case of mall counts within the country, the distant context impelled students to split the problem into more sensible units (i.e., number of malls in big cities and small cities). Hence, the close-proximity on the Student Weight Problem supported the sensemaking-facilitator affordance while the distant-proximity on the Mall Count Problem produced the strategy-promoter affordance in making model-based estimates. Other studies also support the former affordance in that the closer the context of the Fermi problems, the easier and the more authentic estimations the students could make (Albarracín & Gorgorió, 2014; Albarracín et al., 2021; Ärlebäck, 2011).

The latter affordance – strategy-promoter for model-based estimation – is new to the literature although splitting strategy is not new (c.f. Albarracín & Gorgorió, 2014). More specifically, what is novel in the current study is the interaction between the distant-proximity of the problem context and the splitting strategy, which is important for two reasons. First, creative ways of thinking that are comfortable with the uncertainty of the nonroutine problems signify high problem-solving skills (Güner & Erbay, 2021), which was the case in distant-proximity problem context in this study. Second, the reverse interaction that the Fermi problem at a closer proximity was more attainable for students due to the alignment with their personal knowledge raises a concern for mathematics educators to pay attention. On the one hand, the proximity of the closeness of the context should not make the solution trivial, which would violate the open-ended nature of the Fermi problems (Ärlebäck, 2009). On the other hand, the proximity of the closeness of the context should not be too far. In this regard, there seem to be two ways of solving problems in distant context. The first is to generate specific strategic thinking for that context. This simple approach would not incorporate the elements of complexity needed to fit the final model to the problem situation (Albarracín et al., 2021). The second method is to solve the close-context problem and scale up the solution to the distant-context problem, as suggested in upscaling pedagogy (Albarracín & Ärlebäck, 2025; Albarracín et al., 2022). To execute this way, students must be able to find a variable or a set of variables that allows them to establish the comparison between the chosen near context and the far context provided by the problem, and this is not an obvious decision.

Close-proximity context provides reference tools for model-based estimation

Group 3 and Group 4 produced structurally similar model-based estimation in both problems. Specifically, on the Student Weight Problem, their model-based estimation structure was splitting the problem into four simpler problems, which enabled making estimations in a smaller range (i.e., weights of students at each grade level) and then combining those to get a more reasonable final estimate for the problem. On the other hand, on the Mall Count Problem both groups' model-based estimation indicated proportionality to scale up (i.e., multiplication of two estimates: the number of provinces in Türkiye and the number of malls in each city) although the number of provinces was not needed to be estimated if students had factual knowledge. While Group 3 knew that there are 81 provinces in Türkiye, Group 4 did not, and so produced the estimate of 92 for the number of

provinces. Given that each student (or each group of students) has a different knowledge relationship with the contexts provided (Ärlebäck, 2009), each student has to deal with diverse views of reality and different strategies that are provided by the members of each working group, which intrinsically promotes flexibility in the solution. However, we also observed that both Group 3 and 4's model-based estimations on the Mall Count Problem were not as detailed as in the first problem. A possible reason for their deeper engagement in the Student Weight Problem could be the closeness of the context and not requiring students to imagine a large-scale quantity as in the case of Mall Count Problem. In other words, "weight" as a quantity was more sensible in students' own circumstances, so they could think of different options (i.e., sensemaking-facilitator and estimation-referent identifier affordances of the context proximity). Therefore, we claim that the model-based estimation developed in a distant-proximity context did not involve as much convolution as the ones in a close-proximity context, which is similar to other researchers' observations (e.g., Albarracín & Gorgorió, 2014; Albarracín et al., 2021, 2022).

In the Student Weight Problem, Group 5 developed a simple strategy involving only one multiplication statement but not a simplistic one in the sense that the multipliers were gathered with systematic reasoning (i.e., iteration of a unit strategy). However, their algebraic way of thinking on the Mall Count Problem did not indicate a reasonable solution. Similar to other groups, for Group 5, the larger-scale estimation and the distant-proximity of the context in this problem produced more struggle in the Fermi situation. Students struggled to scale the number of malls in their cities to the whole country, which might involve many factors such as population, area of the city, average wealth rate of the city, etc. Therefore, the iteration of a unit strategy, used in the close-proximity context, was not efficient in the distant-proximity case. Thus, the closeness of the context and attainability of the size of the quantity were found influential (i.e., sensemaking-facilitator and estimation-referent identifier affordances of the context), which confirmed the related studies (Albarracín et al., 2022; Pla-Castells & Ferrando, 2019).

Lastly, we observed that four groups (i.e., Group 3, 4, 5, and 6) on the Student Weight Problem and two groups (i.e., Group 2 and 6) on the Mall Count Problem used the splitting strategy. This model-based estimation strategy has also been a common – basic but efficient – in modeling Fermi problems in other studies (e.g., Albarracín & Gorgorió, 2014). In fact, majority of the groups' using this strategy in the close context proximity case confirms our aforementioned claims. That is, the more students related the context to their lives, the more they could attain the size of the quantity in reference to their own circumstances. Furthermore, the narrower the quantity, in terms of the size, involved in Fermi situations, the more sophisticated thinking the students could engage in.

Concluding remarks

This study concludes that the range between close-proximity and distant-proximity of the problem context needs to be carefully considered by mathematics educators because the entire range may not be easily sensible and engageable by students. Herein, we refer to *a zone of the context proximity of the problem that is attainable by students to develop a model-based estimation*. This zone explains the potential sense-making situations in which students would demonstrate strategies for developing a model to make reasonable estimation. Furthermore, this zone of context proximity embraces some affordances in development of model-based estimation. We, in this study, found sensemaking-facilitator, estimation-referent identifier, and strategy-promoter affordances in shaping students' ways of thinking on the Fermi problems.

These conclusions are informative and illuminating not only in mathematics education but also for STEM education where the contextual aspect of the problems has a critical role in problem solvers' modeling-based and engineering-based thinking (Ärlebäck et al., Ärlebäck et al., 2024; Karatas-Aydin & Isiksal-Bostan, 2023) and creative thinking (Lunt & Helps, 2001; Sitorus & Masrayati, 2016). As our findings highlight the importance of the context proximity of Fermi problems, we suggest using problems at both close and distant proximities and considering the idiosyncratic affordances

associated with different context proximities (e.g., sensemaking-facilitator, estimation-referent identifier, and strategy-promoter affordances). In this regard, our study connotes potential instructional design in STEM education; that is, sequencing the STEM activities considering the proximity of the problem context. Considering the affordances identified in this study, the STEM problems may be sequenced from the close-proximity problem context to distant-proximities. Such scaffolding in STEM education with a sequence of activities may foreground contextual aspects of STEM problems as well as disciplinary aspects (i.e., mathematics, science, engineering, and technology). This approach may further promote the development of reasonable, relevant, and creative strategies which are essential for the next-generation STEM-related occupations.

Limitations and recommendations

This study indicated that the proximity of the problem context afforded the development of model-based estimation in Fermi situations. However, the findings are limited to 12 sixth-grade students working in pairs on only two Fermi problems in an online platform. Hence, the data analysis of two Fermi problems in this study confines our understanding of the aspects of the model-based estimations related to within/beyond-direct-reach of students' lives and reference tools and restrains us to distinguish the aspects related to close versus distant proximity of the problem context. Therefore, we suggest increasing the number of Fermi problems considering these two, and even more, characteristics of the context proximity. We foresee the potential contribution of studies which explore the larger number of students' ways of thinking in a larger set of Fermi problems varying in the characteristics of the context proximity. We believe that such studies may not only consolidate but also extend the findings of the present study. In addition, although how the online platform influenced students' work was beyond the scope of this study, we suggest future research to investigate the characteristics of the problem-solving environment (i.e., online, in-class, outdoor, etc.) on students' estimation.

The findings of the study also revealed differences in groups of students. For instance, Group 1 performed lower in both Fermi problems. This may be due to individual differences of students regarding their mathematics achievement. In this study, we could not relate students' achievement and their estimation-based thinking, which constitutes another limitation of the data set. In this regard, we suggest future research considers students' mathematics achievements in schools and further examine how the proximity of problem context functions differently for different achievement levels and what other affordances may appear differently for low- and high-achieving students.

Regarding the implications for mathematics teacher education, the findings of the study suggest that while selecting and sequencing the estimation-based Fermi problems in classroom use, teachers, teacher educators, and researchers should consider the proximity of the problem context, particularly whether the problem context is within the students' attainable zone. This requires a repertoire of teacher knowledge and skills that are built on empirical research, such as the current study. Still, the literature demands studies investigating how to train mathematics teachers for examining the characteristics of the problem context attainable by students and developing a practice of task selection and sequencing within the targeted zone of context proximity.

Notes

1. This study presents the reanalysis of the data collected for a master's thesis (i.e., Arıcan, 2022).
2. While there may not be subtle individual differences in students' academic achievement within pairs, we observed considerable disparity between pairs.

Disclosure statement

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Authors' contributions

The data of this study was collected by the third author for her master's thesis. For this study, the first and second authors reanalyzed a portion of the dataset and were the main researchers organizing and writing this manuscript.

Availability of data and materials

The data analyzed for this study was archived by the third author under regulations of the Institutional Ethics Committee Approval, and upon request, only written data can be provided anonymously. Audio and video data including personal information were archived in safe drives based on the regulations of the Institutional Ethics Committee Approval.

Declaration of AI-assisted technologies

During the preparation of this work, the author(s) did not use any AI-assisted technologies to produce the analysis or the text of the manuscript. Yet, using Grammarly, the author(s) reviewed and edited the content as needed and take full responsibility for the content of the published article.

Ethical Approval

For ethical considerations, Institutional Ethics Committee approval was received. Based on the regulations of the Institutional Ethics Committee Approval, the participation to this study was voluntary-based and informed consent, and parental consent forms were received from the participants prior to the data collection process.

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