Chapter 12
Lessons Learned from Research on Fermi Problems and Mathematical Modelling: Theoretical and Practical Implications for School STEM Education

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Abstract This chapter aims to demonstrate how adopting a mathematics education research perspective on Fermi problems can contribute to and enhance the practice and understanding of STEM education. To achieve this, we first discuss mathematical modelling from a mathematics education perspective and then briefly elaborate on modelling as discussed in STEM education. Our attention is then directed towards the inherent coupling of Fermi problems to mathematical activities and contents of the mathematical curriculum, in particular, estimates, estimation and mathematical modelling in mathematics education research. We present a framework for connecting recent advances in the research on Fermi problems to teaching and learning in STEM education and discuss the example of *The tin foil boat challenge*.

Introduction

The STEM subjects—Science, Technology, Engineering and Mathematics—which have been brought together as an integrated educational research discipline (Kelley & Knowles, 2016), have gained both interest and prominence in educational research over the last 20 years (Li et al., 2020). Although there are different interpretations in the literature of what the acronym *STEM* and *STEM education* entail (Honey et al., 2014), Li et al. (2020) argue, based on their review of literature published between 2000 and 2018, that there is increasing clarity of terminology and a converging understanding of the identity of STEM education in the research field. However, as noted by many researchers (e.g. Fitzallen, 2015; Maass et al., 2019), the role and

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function of mathematics in STEM education is often invisible. In many cases, instead of STEM activities promoting the exploration of the uses of mathematics in rich contexts, the use of mathematics is reduced to serve as a tool in the service of other disciplines. Therefore, from the perspective of teaching and learning mathematics in the context of STEM education, a major concern is whether mathematics is taught and learned only incidentally, when mathematical knowledge and competence are needed in the application of mathematics in situ, or whether it is considered more fundamental in integrated STEM education and requires explicit and systematic attention (Anderson et al., 2020).

Hallström and Schönborn (2019) as well as Hjalmarson et al. (2020) have discussed how *models* and *modelling* are key concepts in STEM education. The use of these concepts shows some similarities (as well as some differences) across STEM disciplines. These authors argue that models and modelling could serve as a bridge between STEM disciplines in integrated STEM education practices. This argument is consistent with the findings of a literature review by Ärlebäck and Albarracín (2019) on the use of Fermi problems (FPs) in the STEM disciplines. Two examples of FPs are *How many liters of water does one person use in a year?* (Okamoto et al., 2023) and *How many cars can fit in the faculty parking?* (Segura & Ferrando, 2021). FPs have been characterised as contextualised miniature modelling problems (Robinson, 2008; see below for further elaboration), and the findings of the literature review by Ärlebäck and Albarracín (2019) illustrate how the use of FPs in educational settings can act as integrators of STEM disciplines as well as facilitators of learning content within one or more STEM disciplines. In addition, recent research on the use of FPs in mathematics education highlights how this problem type can connect to and make mathematical curricula content explicit to teachers and learners (Albarracín & Ärlebäck, 2019; Ärlebäck & Albarracín, 2022). Taken together, and supported by the established field of research on modelling in mathematics education (Blum, 2015; Leung et al., 2021) that focuses on modelling both as a mathematical curriculum goal in its own right and as a vehicle for learning other mathematical curriculum content (Julie & Mudaly, 2007), this suggests that a viable strategy for making the role, function and importance of mathematics in STEM education more visible and explicit may be to use FPs and adopt a modelling perspective on STEM education.

This chapter aims to demonstrate how adopting a mathematics education research perspective on FPs can contribute to and enhance the practice and understanding of STEM education. To achieve this, we first discuss mathematical modelling from a mathematics education perspective and then briefly elaborate on modelling as discussed in STEM education. Our attention is then directed towards the inherent coupling of FPs to mathematical activities and contents of the mathematical curriculum, in particular, estimates, estimation and mathematical modelling in mathematics education research. We connect recent advances in the research on FPs to teaching and learning in STEM education and discuss the example of the FP *The tin foil boat challenge.*

Mathematical Modelling

Just as the words models and modelling are used with different meanings and interpretations in everyday life, many different approaches and perspectives to mathematical modelling can be found in the mathematics education research literature (Kaiser & Sriraman, 2006). In this chapter, we want to use a broad and inclusive understanding of mathematical modelling in order to connect the research in mathematics education on modelling to the general discussion of modelling in the other STEM subjects. Hence, we draw on Niss et al. (2007) and consider

- That a *mathematical model* can be seen as consisting of a triplet including an extra-mathematical situation or domain, a mathematical domain, and a mapping connecting the extra-mathematical and mathematical domains; and
- That *mathematical modelling* can be seen as "the whole process of structuring, generating real-world facts and data, mathematising, working mathematically, and interpreting/validating (perhaps several times around the loop)" (9–10).

From this broad perspective, mathematical modelling therefore essentially involves identifying, delimiting and selecting both the non-mathematical and mathematical domains relevant to a problem situation, and specifying the mapping between the two.

As noted by Niss (2015), a modelling process that results in a mathematical model can serve both descriptive and prescriptive purposes. In descriptive modelling, mathematics is used to describe and represent an existing situation or phenomenon in order to gain insight, provide explanations or predictions of real-world aspects. Prescriptive modelling, on the other hand, concerns informing decision-making or action in order to influence or organize (future) real-world scenarios. During both descriptive and prescriptive modelling, finding a suitable mathematical domain that can map and connect to the extra-mathematical domain of the situation or phenomena is crucial to achieve useful results. The proficiency and comprehension of the modeller in mathematics are key factors for success. In other words, in mathematical modelling the extra-mathematical situation or problem encourages and motivates the exploration, teaching and learning of useful and applicable mathematical content and topics. For the mathematical models, both descriptive and prescriptive, to effectively and efficiently connect to the extra-mathematical domain, it is crucial that the modeller has a deep understanding of both the problem situation at hand and the extra-mathematical domain involved. In other words, mathematical modelling also encourages, supports and motivates the teaching and learning of the knowledge and skills in the extra-mathematical domain. Regarding models and modelling in mathematics education research, Julie and Mudaly (2007) have discussed a similar duality in the teaching of mathematical models and modelling. According to Julie and Mudaly (2007), mathematical models and modelling as a mathematics curriculum content can be regarded both as a goal in its own right and as a vehicle for learning other mathematical content. In this chapter, we use and expand on Julie and Mudaly's (2007) concept

of dual use to encompass not only mathematics but also other STEM subjects as the extra-mathematical domain within the modelling process.

The Models and Modeling Perspective

Ärlebäck and Albarracín (2023a, 2023b) argue that *the models and modeling perspective* (Lesh & Doerr, 2003) is one perspective in mathematics education research that effectively encompasses the teaching and learning of mathematical modelling as both content and vehicle. According to this perspective, models are considered to be human constructs and conceptual systems that are inherently social in nature. They aid in making sense of problems, situations and phenomena. Models are systems composed of elements, relationships, operations and rules that can predict, explain or describe the behaviour of another system. From this perspective, learning fundamentally involves creating and developing useful and generalized models. Therefore, in teaching and learning, the role of modelling activities is to enable students to develop models by involving them in the creation, development, understanding, modification, and use of their own models, in order to make sense of various situations and contexts (Lesh & Doerr, 2003).

Compared to the above more general description of mathematical models and modelling based on Niss et al. (2007) and Niss (2015), the models and modeling perspective (Lesh & Doerr, 2003) provides a more practice-oriented framework for working with models and modelling in classrooms. This is evident in mathematics education research (see for example Lesh and Doerr (2003)), but it has also been used and explored outside of mathematics education in science and engineering education (e.g. Diefes-Dux et al., 2006; Hjalmarson et al., 2006, 2008). In this chapter, we build on and contribute to this line of research and argument by focusing on the use of FPs as miniature modelling problems (Robinson, 2008), as the characteristics of FPs bear a strong resemblance to the characteristics the three types of modelling activities used in the models and modeling perspective (Ärlebäck, 2009). Considering that we want to broadly situate and elaborate on FPs as modelling activities in STEM education, we next briefly discuss how modelling is used and understood in the STEM education context.

Mathematical Modelling in a STEM Education Context

According to Hallström and Schönborn (2019), there is a growing consensus in STEM education research that it is important for STEM teaching and learning to be authentic in order to be successfully implemented. Authentic in this context is understood to mean that activities and problems should "reflect the natures of the parent disciplines as much as possible" (Gilbert et al., 2000, p. 3). With regard to the notions of models and modelling in STEM education, Hallström and

Schönborn (2019) compared and reviewed how models and modelling are defined, what epistemic roles and functions models and modelling have, and how models and modelling are used and come to the fore in educational practices in the STEM disciplines. Their findings show that models and modelling: (1) bridge the gap between the STEM disciplines and facilitate a more integrated STEM education; (2) promote STEM literacy and facilitate the transfer of skills and knowledge between the STEM disciplines; (3) serve as a route to a more authentic STEM education; and emphasise that (4) models and modelling need to be taught rooted in the STEM disciplines. Hjalmarson et al. (2020) argue along the same lines, and additionally discuss the notions of models and modelling as typically used in computational thinking, and the common theme of general problem-solving competencies.

In Science, models and modelling are fundamentally "always conceptual in nature" (Hjalmarson et al., 2020, p. 226) and are often used to develop descriptions and explanations of natural phenomena (Hallström & Schönborn, 2019). In terms of Niss' (2015) terminology, this emphasis on models and modelling in Science is primarily in line with the mathematical aspects of descriptive models and modelling. In Engineering, on the other hand, models and modelling are used in design processes to generate prototypes or test initial designs (Hjalmarson et al., 2020) or to analyse existing solutions to engineering problems (Hallström & Schönborn, 2019) in order to improve the solutions. Thus, the mathematical aspects of models and modelling in engineering can be said to be primarily prescriptive in nature (cf. Niss, 2015). Although a simplified generalisation, the main difference between the use and function of models and modelling in Science and Engineering is summarised by Hjalmarson et al. (2020): "Where science models the real world, engineering uses models to design for the world" (p. 228).

As a STEM discipline, Technology displays a considerable amount of variety in what it encompasses and how it is comprehended (Power, 2018). Drawing on Honey et al. (2014), Ellis et al. (2020) recognized the following four different perspectives on Technology in STEM: (1) as a product of engineering; (2) as educational or instructional technology; (3) as a tool used by practitioners of Science, Engineering and Mathematics; and (4) as coding or computational thinking. The wide variation in how Technology is used in and related to mathematics (cf. Hansson, 2020) and the other STEM disciplines (Ellis et al., 2020) makes it difficult to characterise the general role and function of the mathematical aspects of Technology in terms of Niss' (2015) notions of descriptive and prescriptive modelling. However, in Hjalmarson et al.'s (2020) discussion of models and modelling across STEM disciplines, Technology is concretised in terms of computational thinking. Based on Shute et al.'s (2017) literature review and the resulting six facets of computational thinking, Hjalmarson et al. (2020) suggest that "[s]tudents' processes of designing, creating, evaluating, and using models as CT [computational thinking] practices can provide a common language for connecting problem-solving across the STEM disciplines" (p. 229).

In this chapter, we take a mathematical perspective on STEM education, understanding STEM education as a general integrated curriculum approach in which mathematics is linked to one or more of the other STEM subjects. We in particular focus on mathematical modelling as a process that connects mathematical knowledge

with the real world, and as such integrate mathematical activities and well defined mathematical learning objectives with one or more other STEM disciplines. Given the broad characterizations of modelling and the mathematical aspects of Science, Engineering and Technology discussed above, we next discuss research on FPs to argue how the role of estimation and FPs can be used as a way of transferring research advances on FPs as mathematical modelling activities in mathematics education to STEM education activities. We provide a framework that facilitating making explicit the connections needed to enable this transfer of knowledge across disciplines.

Estimates, Estimation and Fermi Problems

An *estimate* is an approximate numerical value that quantifies a given quantity and is often used in everyday life as part of on-the-spot decision-making. For example: *Do I need to buy more flour for tomorrow's baking according to this recipe, or will the amount of flour left in the packet be enough?* or, *Given that the news has reported a traffic jam nearby, do I need to leave 10 min earlier than usual to get to work on time?* The general process and reasoning behind an individual making an estimate is called estimation. Estimation is used not only in everyday life, but also in various forms in many scientific disciplines when faced with the need for numerical values of quantities for which no specific methods or reliable data are available, when an accurate calculation is not needed, or is too costly to perform, such as estimating the global carbon budget (Friedlingstein et al., 2019).

Traditionally, estimation in mathematics education has been discussed in terms of computational estimation, numerosity and measurement estimation (Hogan & Brezinski, 2003). However, mathematics curricula in various countries, including the UK, Greece, and Turkey (Andrews et al., 2022), include a more general type of estimation activity that brings the different types of estimation mentioned above to the forefront. These estimation activities involve situations in which the estimators estimate quantities. The estimators, who are the individuals doing the estimation, do this without necessarily having prior developed estimation referents or extensive knowledge of the context for these estimations. It is worth noting that these activities have not been widely discussed in the literature. The type of activity requiring this type of estimation is known as *Fermi problems*. Two examples of Fermi problems (FPs) used in research are:

How many semitrailer loads of bottled water [16.9 fluid ounces] were needed by New Orleans' refugees in the week following Hurricane Katrina? (Taggert et al., 2007).

How many rolls of toilet paper are used in the schools every year? (Ärlebäck & Albarracín, 2023a, 2023b).

Sriraman and Knott (2009) describe an FP as an estimation problem used for the didactic purpose of identifying the initial conditions of the problem and making conjectures based on the various quantities or variables that arise when solving the problem. Characterised as open-ended questions, FPs typically provide little or no

specific information to guide the problem-solving process (Efthimiou & Llewellyn, 2006). This emphasises the need to analyse and think carefully about the problem situation at hand. In an FP, the development of a solution method can be equated with the creation of a model that describes the situation under investigation (Robinson, 2008). An important aspect of FPs from a mathematical modelling perspective is to perform a detailed analysis of the situation presented in the problem statement. The aim is to break down the initial problem into simpler sub-problems, so that a solution can be obtained for the original question through reasoned estimates. Instead of relying solely on reasoned estimates to arrive at a solution, these reasoned estimates can be supported and arrived at by engaging in class activities or problem-solving processes. If these activities and processes are linked to reality, they in addition facilitate validation and critical reflection about the estimates (Albarracín et al., 2022; Sriraman & Knott, 2009). Robinson (2008) characterised FPs as miniature modelling problems partly due to the similarities between the processes involved in solving FPs and more general modelling tasks. Additionally reasons for this characterisation are that FPs as classroom activities typically are more modest in scope, time-requirement, and prior content knowledge specific requirements.

The Fermi Estimation Method

Fermi problems (FPs) are named after Enrico Fermi, a Nobel Prize winning physicist. Fermi used this type of problem in his own research and also included them in his teaching. In one well-documented case, Fermi witnessed the detonation of the Trinity test in the Alamogordo desert in New Mexico and, at the moment of the explosion, scattered small pieces of paper before and during the passage of the shock wave. By measuring the distance travelled by the pieces of paper, he estimated the power of the explosion to be about 10,000 tons of TNT, which turned out to be of the same order of magnitude as the estimate obtained from calculations based on the more robust data provided by the sensors placed in the blast zone (Allison et al., 1955). Fermi utilised this type of problem in his physics classes to highlight the capabilities of inductive reasoning and to equip his students with the knowledge required to undertake laboratory experiments (Robinson, 2008). Even though the use of this type of problem, which was pioneered by Fermi, has spread to classrooms of all levels and even other technological fields, research on the educational applications of FPs has been relatively limited until recent decades when it began to receive more attention. Interestingly, the majority of research on the use of FPs has been conducted in the field of mathematics education (Ärlebäck & Albarracín, 2019).

Several authors have pointed out links between FPs and mathematical modelling (Ärlebäck, 2009; Czocher, 2016; Albarracín et al., 2022; Peter-Koop, 2009; Sriraman & Lesh, 2006). The literature shows that FPs promote mathematical modelling in primary education (Albarracín & Gorgorió, 2014; Henze & Fritzlar, 2010; Peter-Koop, 2009), secondary education (Ärlebäck, 2009; Keune & Henning, 2003) and tertiary education (Czocher, 2016; Ferrando & Segura, 2020). This

research frequently centres on decomposing and breaking down the initial problem into simpler sub-problems, which are solved independently using estimates. This approach is commonly referred to as the *Fermi method* (or *Fermi estimation method*; see Fig. 12.2a and below for further elaboration). Research suggests that students can be successfully taught to use this method appropriately to solve a wide range of FPs through direct instruction (Barahmeh et al., 2017; Raviv et al., 2016). While the majority of research on FPs has been conducted in mathematics education (Ärlebäck & Albarracín, 2019), the Fermi estimation method has also been applied in other disciplines such as economics to identify potential new markets (Anderson & Sherman, 2010), astrobiology to estimate the amount of energy the Earth receives from the Sun (Cordry, 2010), and engineering projects to assess the feasibility of proposed solutions (Shakerin, 2006). Taken collectively, this indicates that utilizing FPs could be a viable strategy for engaging students in integrated STEM educational settings.

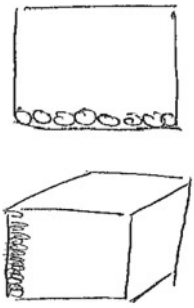
One line of recent research, represented for instance by Albarracín and Gorgorió (2018), has focused on the use of sequences of FPs rather than as stand-alone tasks. These authors focused on initial ideas related to density and used FPs with the contexts of the tasks becoming progressively more general and removed from the students' everyday experience:

FP1: How many people can we fit in the schoolyard?
FP2.1: How many people could we fit into the Palau St. Jordi sports pavilion for a concert?
FP2.2: How many people could we fit in the town hall square during a public protest?
FP2.3: How many people could we fit in Plaça Catalunya square during a public protest?
FP3: How many trees are there in Central Park?

The use of sequences of FPs illustrates the feasibility of using FPs to focus on specific mathematical content and concepts. In addition, it also illustrates that FPs can naturally have various functions in a sequence of tasks, making them adaptable for flexible implementation in integrated STEM tasks that typically involve sequences of tasks belonging to different STEM disciplines.

To illustrate the Fermi estimation method and the way in which a problem is decomposed into related sub-problems, we will use the problem *How many coins fit into a 1 m3 cubic safe*? discussed and analysed in Albarracín and Gorgorió (2014). One of the students in their study developed the following proposal to solve this FP (Fig. 12.1).

By using the cylindrical shape of the coins, the student proposed solving the problem by assuming that the coins could be arranged in a three-dimensional grid distribution. Consequently, the student aimed to solve the following two sub-problems: *How many coins can fit into one of the dimensions of the base of the safe?* and *How many coins can be stacked on top of each other to cover the height of the safe?* Fig. 12.2b illustrates how these two sub-problems are intended to answer the original problem.



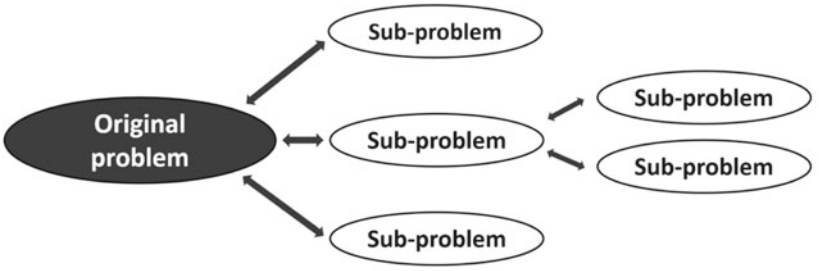
I would calculate the diameter of the coin, so I would know how many coins fit in the base by dividing a meter by this diameter (call the result of this calculation X).

I would then find the thickness of the coin and divide a meter by the thickness (call the result of this calculation Y).

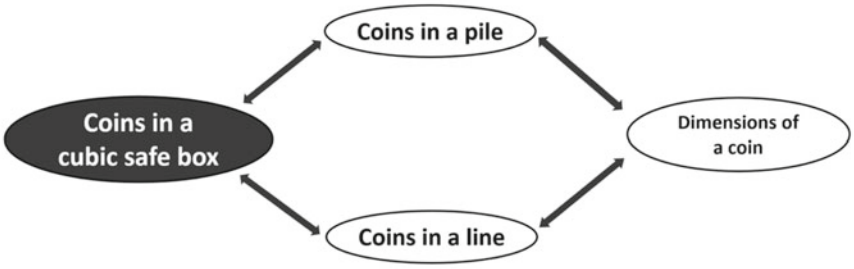
Then what I would calculate next would be: $X^2 \cdot Y$

Then you would have the number of coins that fit in a cubic meter.

Fig. 12.1 A student's proposed solution to the FP *How many coins fit in a 1 m^3 cubical safe?*



(a)



(b)

Fig. 12.2 **a** A generic illustration of the Fermi estimate method; **b** The Fermi estimate method illustrated on the FP *How many coins fit in a 1 m^3 cubical safe?*

Fermi Problem Activity Templates

To support teachers in getting their students to work on FPs in the mathematics classroom Albarracín and Ärlebäck (2019) have introduced *Fermi Problem Activity Templates* (FPATs) to explain and facilitate the use of FPs and the Fermi method. Similar to the Fermi estimation method illustrated in Fig. 12.2, the FPATS are graphical representations that show the structure of the problem broken down into sub-problems. Furthermore, a FPAT also shows the mathematical activities that are used to, or suggested to be used to, determine the necessary quantities required to obtain an answer to the problem. An FPAT focuses on the structure of the solution to the problem and the model developed to solve the problem, and can be used as both a didactical tool or as a tool for analysis. As a didactic tool to it can represent a problem's solution, derived from an a priori analysis, in terms of the sub-problems that need to be solved. As an analytical tool FPATs depict the structure of students' activities when solving a problem. It's worth noting that each FP can be approached differently, depending on the students' extra-mathematical knowledge at each educational level (Ärlebäck & Albarracín, 2023a, 2023b). Thus, an FPAT only characterizes a single approach to the problem's solution, and should not be thought of as a static recipe or algorithm to blindly follow to solve the problem. One significant characteristic of an FPAT is that it broadens the conventional emphasis of FPs on estimation, with other mathematical activities available for obtaining the necessary numerical values of quantities to address a problem. As a result, work on FPs in the classroom can be linked and honed towards specific mathematics curriculum content and topics with greater ease.

In order to explicitly visualise the structure of a solution to a FP and the different types of mathematical activities that can be used to solve it, the FPATs employ different graphical representations for four mathematical activities associated with different contents and parts of the mathematics curriculum. The four mathematical activities, which were identified in an exhaustive review of the literature on the use of FPs in the STEM disciplines (Albarracín & Ärlebäck, 2019), are the following:

(a) *guesstimation* (providing a fast answer as in traditional FP solving);
(b) *measurement and experimentation* (of relevant values, in the lab or the field and using tools);
(c) *looking up data* (in official databases or online newspapers); and
(d) *polling or statistical data collection* (using surveys and questionaries, etc.).

To provide an instant overview of the solution, each activity in an FPAT is represented by a specific symbol or shape, according to the following scheme: guessing (ellipse), experimenting (trapezoid), looking up data (rectangle) and polling or collecting statistical data (hexagon). In Fig. 12.3, an FPAT depicts the student's solution to the earlier problem of determining the number of coins that can fit into a cubic safe measuring 1 m^3. In Fig. 12.3, the FPAT illustrates: (1) the two sub-problems identified by the student (denoted by the two inner square brackets); (2) what activities are proposed to obtain the required numerical values (represented by the trapezoids—measurement in both sub-problems); and (3) how the sub-problems and their

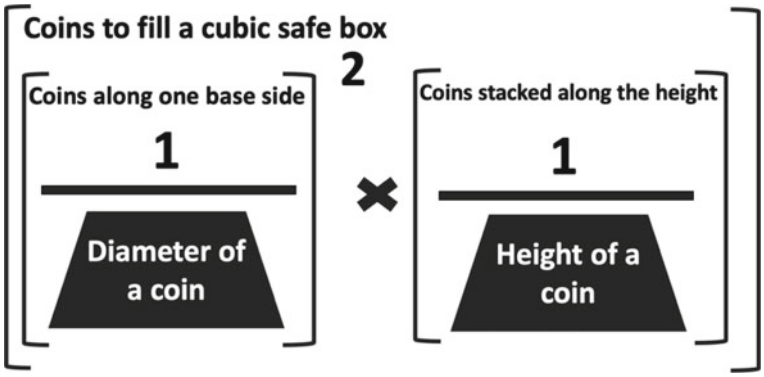


Fig. 12.3 A FPAT for the coins in safe box problem

respective results are related to the original problem in order to provide an answer to the posed question expressed as a formula (here capturing the three-dimensional lattice distribution arrangement of the coins in the safe model).

FPATs are a versatile tool. They have been used for research purposes to identify and characterise diverse approaches to the same problem, thereby facilitating comparison of students' work (Albarracín & Ärlebäck, 2019). In addition, FPATs can be useful in classroom activities. On the one hand, research indicate that FPATs can support students in their individual work in classrooms or during group discussions by serving as a tool for clarifying complex ideas to their peers and teachers (Ärlebäck & Albarracín, 2023a, 2023b). On the other hand, by conducting an a priori analysis of an FP and FPAT has been shown to assist pre-service teachers in designing and preparing classroom work (Ärlebäck & Albarracín, accepted). FPATs can aid teachers at various educational levels in creating classroom tasks related to predetermined learning goals or specific curriculum content, comparing methods, and engaging with the progressively intricate facets of problem-solving and mathematical modelling, which are crucial challenges in the instruction and acquisition of mathematic.

We now pull together the issues as aspects discussed about by situating and elaborating on FPs and FPATs from a modelling perspective in a STEM education context.

Fermi Problems: Integrators and Facilitators in Learning STEM Disciplines

The literature review on the use of FPs in STEM by Ärlebäck and Albarracín (2019) focused both on the arguments invoked for using FPs in educational settings, and on the categorisation of research findings involving FPs. In terms of the arguments

used for using FPs, four main reasons were identified: (i) *estimation as an important skill and tool*; (ii) *promoting (critical) understanding and importance of quantities within the disciplines*; (iii) *facilitating and practising general problem-solving skills*; and (iv) *promoting, learning and engaging in modelling*. Here, (iii) in particular is consistent with Hjalmarson et al.'s (2020) argument that the use and development of general problem-solving skills is a unifying and common theme across the STEM disciplines. In educational settings, both (i) and (ii) are rooted in the nature of the parent disciplines and in this sense resonate with Hallström and Schönborn's (2019) emphasis on authenticity. Reason (iv) on the other hand supports the idea that FPs are emerging as a novel tool and a pathway towards more integrated STEM disciplines, as suggested by both Hallström and Schönborn (2019) and Hjalmarson et al. (2020). Indeed, the latter point is further emphasised in the conclusions of the review by Ärlebäck and Albarracín (2019), namely that FPs—in their capacity as miniature modelling problems—and FPATs have both the practical and theoretical prerequisites to act as integrators between STEM disciplines. A second result of the literature review was the function of FPs as facilitators of learning in the STEM disciplines. In this context a FPAT has the capacity to provide a map over the sub-problems involved and explicitly show the interplay and relationship of the different relevant STEM disciplines coming to the fore as well as are integrated in the problem at hand. In particular, the FPAT facilitates (1) emphasising the role and function of Mathematics in solving the problem; and (2) explicitly linking the mathematics used to the mathematics curriculum content. Therefore, a well-designed FP can serve as a starting point for teaching and learning in a STEM educational setting focused on different disciplines, as demonstrated by Hansson (2020) in relation to technology, Robinson (2008) in relation to science, and Shakerin (2006) in relation to engineering.

Overall, the preceding discussion supports the view that FPs and FPATs are capable of promoting interdisciplinary integration within STEM education, which confers benefits both to teaching and learning both in the individual STEM disciplines and in integrated STEM education. In particular, a FPAT has the potential to explicitly highlight the important roles and functions of Mathematics in the solution of a FP in relation to mathematical curricula content. Next, we explore and elaborate on this capacity of FPs and FPATs by discussing the activity called *The Tin Foil Boat Challenge.*

The Tin Foil Boat Challenge

Building on the discussion in the previous sections, we now demonstrate the potential of FPs and FPATs in facilitating the integration of STEM disciplines as well as highlight the mathematics coming to the fore using a modified version of *The tin foil boat challenge* (Lee, 2014). In this challenge, students design and construct boats from aluminium foil to investigate how their size and shape affect the weight they can carry. The question posed to the students is the following *How many coins does it take to sink a tin foil boat made from a sheet of tin foil measuring 30 × 40 cm?*

This challenge aims to engage students with STEM concepts such as Archimedes' principle (which states that any object immersed in a fluid exerts an upward or buoyant force equal to the weight of the fluid displaced by the object), buoyancy (the ability of an object to float in water or another fluid), material density (the mass of an object per unit volume) and weight distribution (the arrangement or positioning of weight within an object). These concepts are all under the umbrella of Physics (i.e. Science); therefore, this challenge involves some science content. Furthermore, this challenge necessitates the design and construction of a boat that optimises the usage of a tin foil sheet of a given dimension. The boat should be capable of carrying as many coins as possible while considering the arrangement of coins in the boat. This is an engineering problem that entails several variables and calls for a substantial amount of trial and error, which can benefit from estimation (i.e. Mathematics) as a form of anticipation and skill development in the laboratory (Robinson, 2008). Engineers routinely use estimation in their professional practice to provide answers to ill-defined problems or when a detailed solution would be too costly in terms of resources such as money or time (Shakerin, 2006). From a Mathematics point of view, and depending on the grade level in which the tin foil boat challenge is used, different areas and topics related to geometry, optimisations and differentiation can be used and explored.

Since intuitively a tin foil boat full of coins is likely to sink, one concrete way of thinking about this problem is to estimate an upper bound for the number of coins needed to perform the experiment. That is, to ask the FP: *How many coins do we need to fill a boat made of 30 × 40 cm tin foil*?

The solution to this introductory FP can be approached in a number of ways, each represented by a different FPAT. One of the decisions to be made is the shape of the boat, given the constraints of the tin foil provided. From an engineering point of view, the boat must be able to float, transport objects and navigate, which will influence the design of the boat as it must be able to cut through the water (Fig. 12.4, left). From a mathematical point of view, however, it makes sense to first try a design of the boat as a rectangular box, as this simplifies calculations and estimations (Fig. 12.4, right). Thinking about how to find the maximum volume of a rectangular box-shaped boat, for example, offers different opportunities for teachers to link and explore different mathematics curriculum contents and activities, depending on the grade level in which the challenge is implemented. For example, trial and error, building your own designs and then taking measurements, or finding an algebraic function of volume and then either using the method of completing the squares or differentiation. Technology can come into play here in the tools used to take measurements or, for example, as a spreadsheet used to explore the algebraic function of volume to approximate the dimensions that give the largest boat volume. In this example presented here however, we will opt for the following proposal to solve the introductory FP:

- First, the volume of the boat is obtained by approximating it to a rectangular box and measuring its height, width and length.

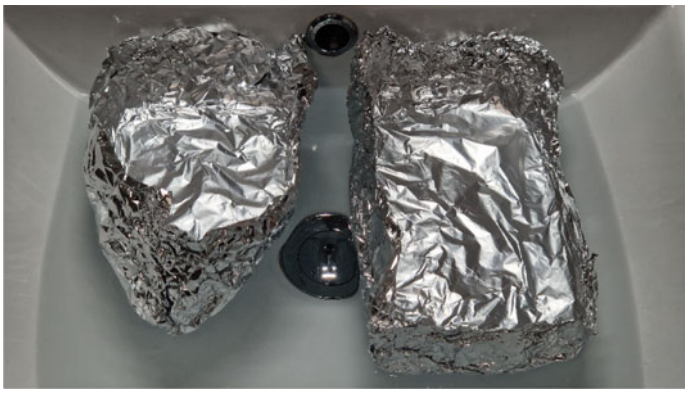


Fig. 12.4 Two different tin foil boat designs

- Secondly, to obtain the total weight of the coins tin boat can carry, the available volume is multiplied by the effective density of the space occupied by a coin, which in turn is calculated by multiplying
  - o the density of a coin which can be found by consulting official sources of the bank issuing the coins; and
  - o the "occupancy factor" derived from observing how much of the area of a square with the same side length as the diameter of the coin is occupied by a single coin.
- Thirdly and finally to get the number of coins that the tin foil boat can hold, dividing the total weight obtained by the weight of a coin. The weight of a coin is obtained by weighing a set of coins and dividing by the number of coins, since we are unlikely to have a scale accurate and precise enough to weigh a single coin.

This process is shown in Figure 12.5 in the form of an FPAT and can be interpreted as a chain of experiments and investigations leading to an answer to the introductory FP: *How many coins do we need to fill a boat made of 30x40 cm tin foil*?

The solution of this introductory FP involves elements from different STEM disciplines. The key elements that affect the total weight of the coins in the boat are the mathematical and structural considerations that come to the fore in the way the coins (cylinders) occupy the design volume of the boat, given the tin foil provided. The FPATs explicitly illustrate how the (mathematical and/or scientific) concept of density emerges naturally, which, as it can be complex for students to grasp, can then be discussed and developed. Furthermore, the FPAT in Fig. 12.5 explicitly provides an accessible and qualitative quasi-algebraic structure of how the quantity of coins necessary to fill a tin foil boat is dependent on the various quantities involved. This representation can be used as a starting point for introducing more formal variables and a continued mathematisation to derive a mathematical formulae. Alternatively, if the aim is to maximize the number of coins one can fit in a thin foil boat, it can be used to engage in a discussion with the students about mathematical strategies

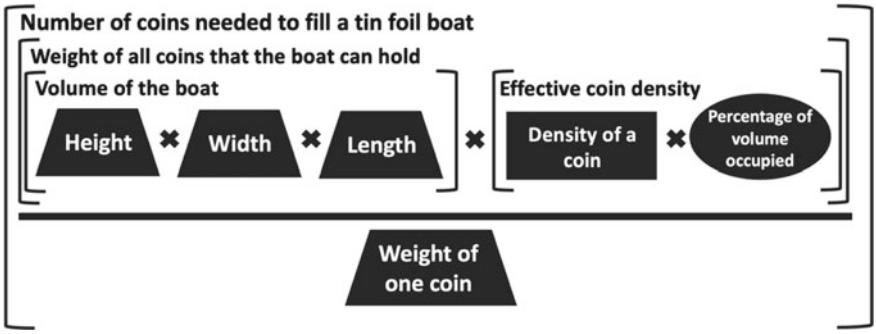


Fig. 12.5 A FPAT for the coins and tin foil boats problem

and principles of designing the boat (its height, width and length) to maximize its volume.

The actual estimate obtained from the introductory Fermi can be used to prepare the materials needed to tackle and experiment with the original problem: *How many coins does it take to sink a tin-foil boat made from a tin sheet of foil measuring 30 × 40 cm?* The introductory FP introduce and prepare the students for this more complex and open-ended question, and to begin to address and understand the scientific concepts of buoyancy and Archimedes' principle. In a classroom setting, the tin foil boat challenge has the potential to generate many different boat designs and results that can be collected for collaborative investigation and analysis. Here, the use of mathematical and statistical tools and representations, created and explored using appropriate technology, facilitates critical investigation and exploration of the scientific concepts and principles involved, as well as drawing conclusions about the 'best boat design' from an engineering perspective.

Brief Summarising Remarks

This chapter discusses how research on FPs can contribute to and enhance the practice and understanding of STEM education. We positioned FPs in relation to modelling in both mathematics and STEM education and contended that FPs can function as integrators of STEM disciplines and facilitators of learning content across one or more STEM disciplines. FPs have proven to be effective as introductory activities to mathematical modelling as well as suitable components of sequences of tasks aiming at developing specific mathematical ideas and content. However, for teachers to productively support their students in this work they require training and support. In this context, we illustrated how FPATs can function as both didactic tools and tools for more systematic analysis. In particular, we argued that FPATs can be used to make the important role and function of Mathematics in STEM education more visible and explicit. In the example of the introductory FP for the tin foil boat challenge, the FPAT

in Fig. 12.5 highlights the mathematical structure of the solution to the problem as well as the importance of understanding how the dimension of the boat affects the available volume.

In our opinion, FPs are potential candidates aligned with the call to use models and modelling to attain more integrated STEM education, as suggested by Hallström and Schönborn (2019) and Hjalmarson et al. (2020). Nonetheless, further empirical studies are required to validate and develop the ideas and framework presented in this chapter. As shown in the literature presented, FPs can play different roles in STEM activities due to their various ties to the STEM disciplines and their accessible nature. A particularly interesting line of future research would be to use the FPAT framework to design FPs that focus on integrating the STEM disciplines. This this is analogous to what is outlined in the tin foil boat challenge, where engineering design meets physical and mathematical concepts, and technology is employed in data collection and analysis. These components intertwine to fulfil the requirement of producing an estimate, which is a key decision-making activity when starting to engage in a STEM project. In activities such as these, the use of FPs and FPATs can assist learners at all levels of education in decision making and understanding complex concepts.

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