Research on cognitive aspects in the solving process of Fermi problems

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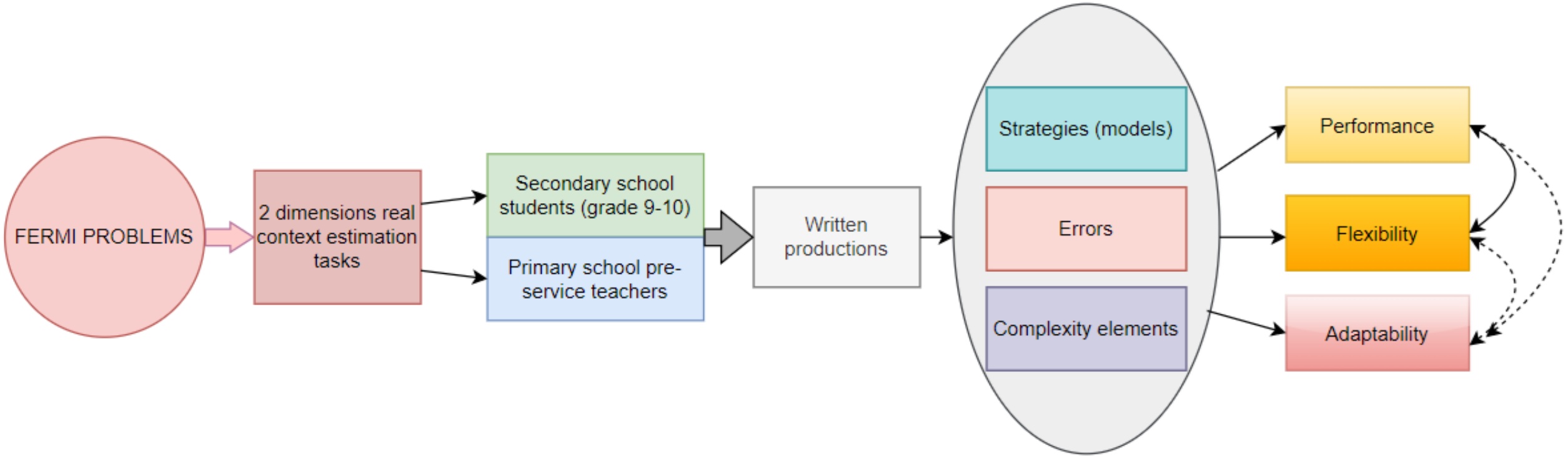
Topic overview



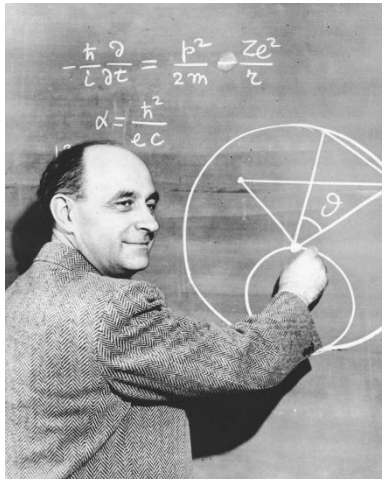

Lluís Albarracín


Carlos Segura


Núria Gorgorió


Marta Pla-Castells

Fermi problems (FPs)



https://www.britannica.com/biography/Enrico-Fermi#/media/1/204747/115395

- Characterized as open questions offering little or no specific information for the problem solvers to direct them in the solution process (Efthimiou and Llewellyn, 2006)
- Some examples: estimating the number of cars on a 3 km tailback on a motorway (Peter-Koop 2009); estimating to within one order of magnitude the number of haemoglobin molecules that your body makes every second (White 2004); and estimating how many shopping malls are in the United States (Anderson and Sherman 2010).
- FPs are those questions that ask students to estimate quantities in the real world by following the *Fermi method* (Arlebäck & Albarracín, 2019)

FP’s & mathematical modelling

We understand mathematical modelling as the problem-solving process encompassing the mathematising of a real-world phenomenon and elaborating a mathematical model to describe, understand or predict the behavior or properties of the phenomenon studied.

Following the definition of mathematical model by Lesh & Harel (2003) as conceptual systems that include the procedures used to create useful constructions, manipulations, or predictions for achieving clearly recognized goals, FP’s can be considered “miniature-modelling problems” (Robinson, 2008; Ärlebäck & Albarracín, 2019).

2 dimensions real-context estimation tasks

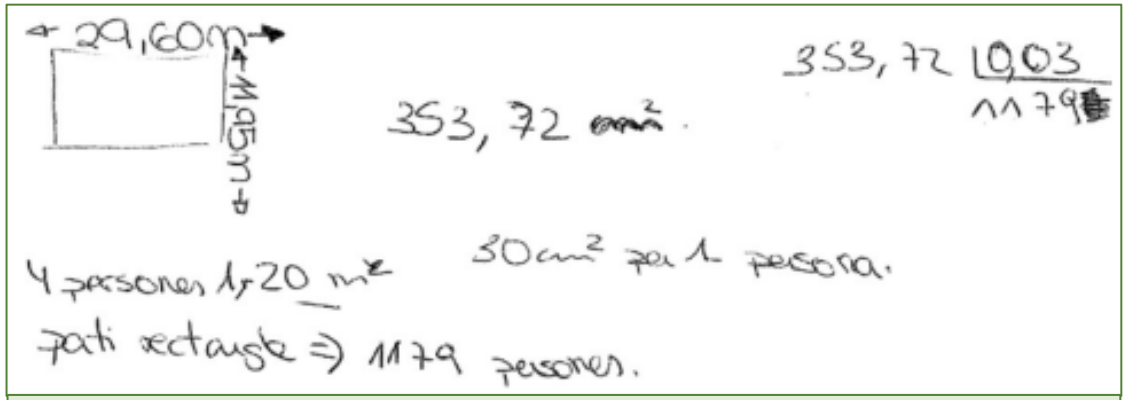
- Context: An end-of-year party will be held in the playground, including a concert, and we have to predict the maximum number of tickets that can be sold.
- Question: *How many people would fit in the playground if we held a concert there?*

- Context: City parks offer residents contact with nature. The quality of the parks depends on their size and, especially, on the number of trees.
- Question: How many trees are there in Central Park?

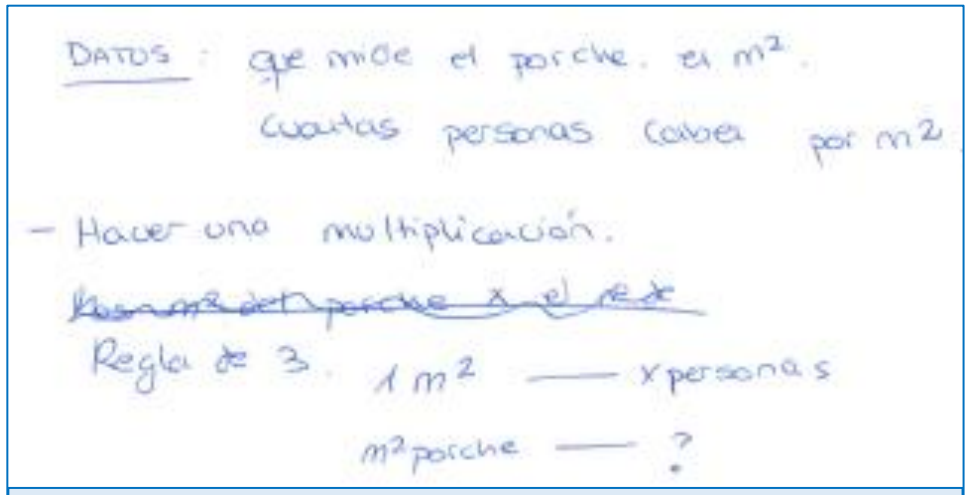
How many students can stand on the faculty porch when it rains?



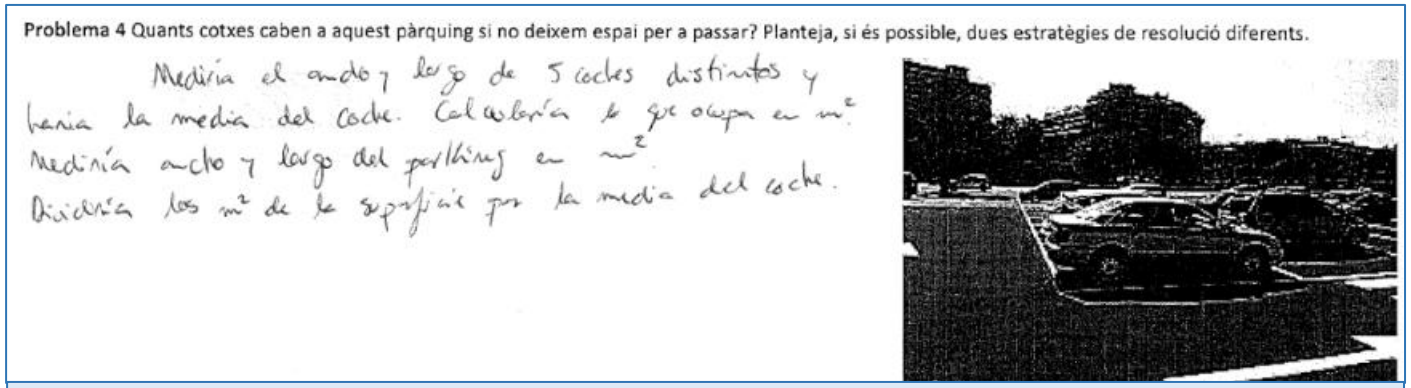
Written productions



Production of a group in 10th grade group of students of Playground Concert problem

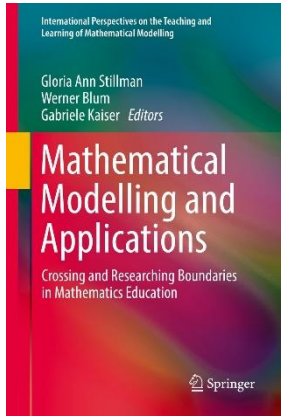


Solution plan for Rain problem. Data: measure of the porch, number of people in 1m^2. “Rule of three” (indeed it is a multiplication)



Solution plan for Cars problems: *I would measure the width and length of 5 different cars and average them. I would calculate the average area occupied by a car, then measure the width and length of the car park in m^2. I would divide that measurement by the average space occupied by a car.*

Description of some preliminary studies with secondary school students' solutions of FPs



Secondary school students (grade 9-10)

Strategies (models)

Chapter 23
Design and Implementation of a Tool for Analysing Student Products When They Solve Fermi Problems

César Gallart, Irene Ferrando, Lluís M. García-Raffi, Lluís Albarracín, and Núria Gorgorió

Abstract In this chapter, we present a tool for analysing the work of secondary-level students from two different schools when they solve a type of Fermi problem. The tool is based on the characterisation of the concepts, procedures and language used to construct the models. Our results show that the proposed tool is useful to describe the models and to distinguish different aspects between the models produced by students without any previous modelling experience and those obtained by students who were already acquainted with working on modelling activities.

Keywords Modelling-eliciting activities (MEA) • Fermi problems • Secondary school • Real life

RESEARCH OBJECTIVES

- To analyse the mathematical models produced by students when solving FPs by using a tool for model characterisation based on Lesh and Harel's (2003) definition of mathematical model.
- To identify the elements that differentiate the models obtained by students with previous modelling experience and those of students that have none.

Secondary school students (grade 9-10) | Strategies (models)

Methological issues

- Participants: 24 10th grade students (seven 3/4-students teams) from an school in Valencia with previous experience with modelling, and a group of 22 10th grade students (six ¾-students' teams) from a school in Barcelona without previous experience with modelling.
- Tasks and experience: Problem A is to be solved in the school playground, and problem B is to be worked on later in the classroom.

Problem A: To organise the school year-end festival, it could be nice to bring up a music band and organise a concert in the schoolyard. In this situation, a good question is: How many tickets can we sell to fill the yard during the concert?

Problem B: How many trees are there in Central Park?[1]

Qualitative analysis of written productions

- For the analysis we relied on Lesh & Harel (2003) definition of mathematical model:
  - **Concepts (mathematical concepts, relationships between them, patterns, etc.)**
  - **Procedures (for data collection about the studied phenomenon and on the mathematical work required to obtain results)**
  - Languages (symbolic, written, sketches, diagrams, etc.)

Results (that will be useful later…)

Secondary school students (grade 9-10) | Strategies (models)

- The analysis of concepts used was helpful for identifying different “conceptual systems” (grid, iteration and density), and different “complex elements” (consider or not unusable space)
- The analysis of procedures was helpful to complete the information about concept (we identified different procedures related to each “conceptual system”)
- This initial tool was useful to identify differences between both groups: students with previous modelling experience tended to formulate models based on more complex concepts, using more rigorous measurement procedures and more elaborate mathematical languages, such as algebraic representations that allow for the generalisation of models. We also observed differences regarding the conceptual systems used in the solution of both problems.

Secondary school students (grade 9-10)

Strategies (models)



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The Role of Context for Characterising Students' Strategies when Estimating Large Numbers of Elements on a Surface

Lluís Albarracín [1] · Irene Ferrando [2] · Núria Gorgorió [1]

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This study focuses on the characterization of the problem-solving strategies that secondary school students (aged 16) developed when required to estimate the number of objects on a surface.

In order to broaden the range of problem-solving strategies, we presented them with a **sequence of tasks** situated in different real-life contexts, but all responding to the same mathematical demand, the estimation of the number of objects.

The tasks given to the students were a type of Fermi problems initially set in contexts close to their own reality, but which progressively became farther removed.

Sequence of tasks

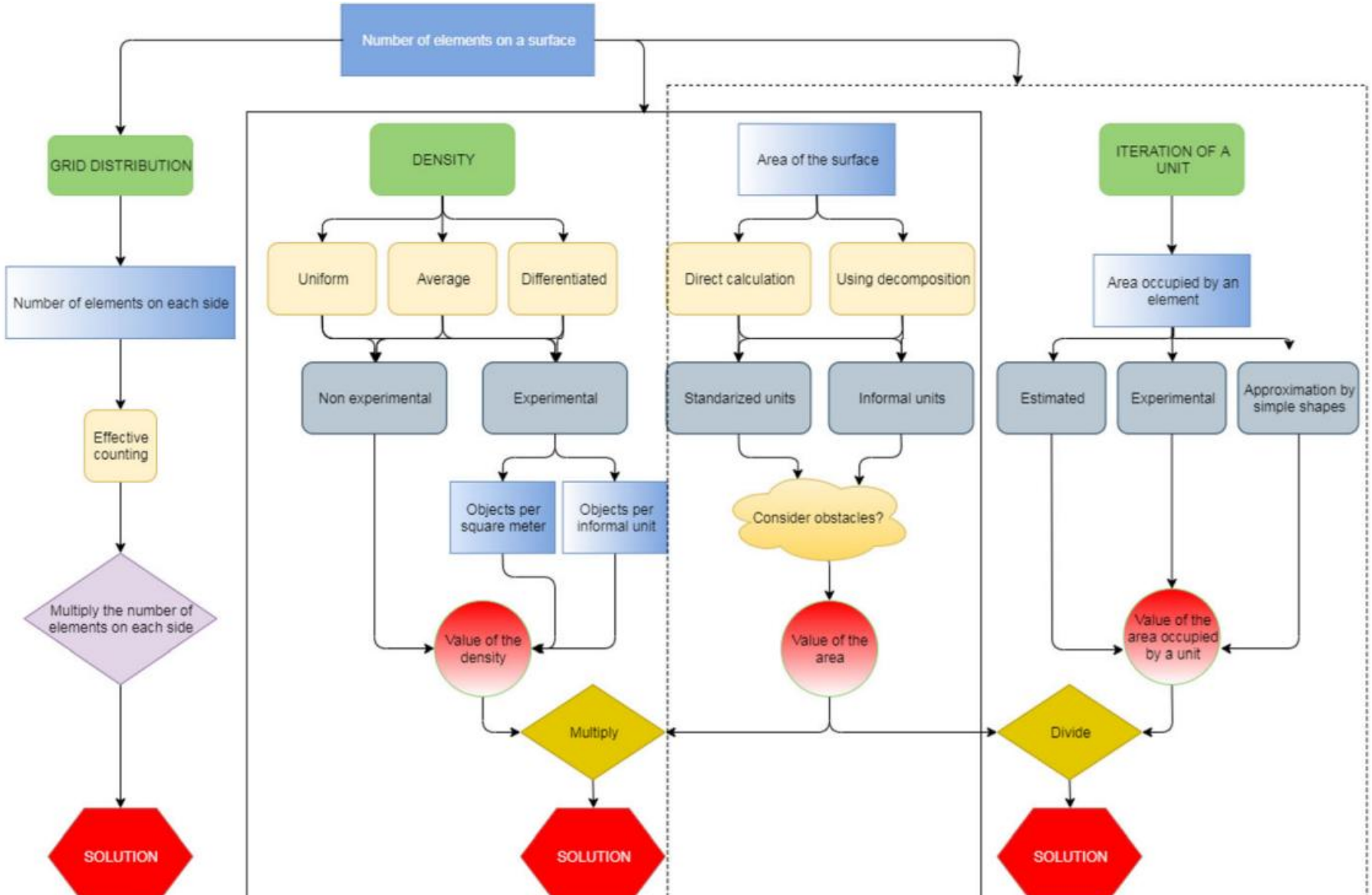
- Task A: Context: An end-of-year party will be held in the playground, including a concert, and we have to predict the maximum number of tickets that can be sold. Question: How many people would fit in the playground[1] if we held a concert there?
- Task B1: Context: Popular music groups use sports halls as venues where they can attract a large audience to their concerts. Therefore, it is necessary to determine their capacity in advance. How many people would fit at a concert held in the sports pavilion of the city where the school is located?
- Task B2: Context: The importance of a demonstration can be quantified by the number of people who support the protest. Question: How many people would fit in the main square of your city if a demonstration were held there?
- Task C: Context: City parks offer residents contact with nature. The quality of the parks depends on their size and, especially, on the number of trees. Question: How many trees are there in Central Park?

Secondary school students (grade 9-10) | Strategies (models)

Data collection and analysis

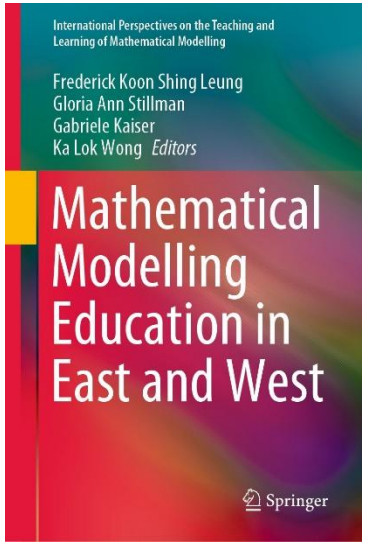
- Participants: 46 10th grade students from two Spanish schools divided into 13 teams of three or four students each.
- Devellopment: The teams worked on task A in a 90-min session, first in the playground, and then in the classroom to write down their procedures and solutions. During the second lesson, which lasted 45 min, the whole class shared and discussed their procedures and results for task A. In the third lesson, which lasted 90 min, students were presented with tasks B1, B2 and C—as a set.
- For each task, we characterized the strategies used by the students, understanding this set of strategies as the solution space generated by all the working groups participating in this study (Leikin, 2007; Leikin & Levav-Waynberg, 2008).

Results: the analysis of solutions for each tasks lead us to build the “tree-like diagram” of strategies



Some insights from these preliminary studies…

- FP's, particularly 2-dimensión real context estimation tasks can be considered as Multiple Solution Tasks
- It seems that there is some relationship between the context features and the solutions
- This tasks may promote the inclusion of "complexity elements" in the solutions
- Since the solution process is closely related with modelling process, these tasks may be useful to study modelling performance
- Since context' features can relate to solution, it may be possible to use a sequence of tasks to study flexibility and adaptability



Primary school pre-service teachers | Strategies (models)

The aim of this study is to confirm the relationship between the characteristics of the context of an estimation task in a real context and the resolution strategies proposed in the solution plans of future primary school teachers.

The reason for working with future teachers is due - from a practical point of view - to the accessibility but we also rely on the fact that subject-related teacher competencies have a strong influence on students' performance (Baumer et al. 2010).

Chapter 10
Analysis of the Relationship Between Context and Solution Plan in Modelling Tasks Involving Estimations

Irene Ferrando, Carlos Segura, and Marta Pla-Castells

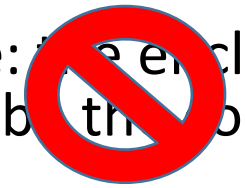
Abstract In this chapter we analyse students' written solution plan of a sequence of modelling tasks involving estimations. Our research objective is to statistically analyse whether there is a relation between the solution plan and the characteristics of the context of the real estimation task. From previous work, we have identified some task variables that are directly related to the context. In this study we have designed a sequence of modelling tasks and we have analysed the productions of $N = 224$ Spanish pre-service primary school teachers. The results show that there is a relationship between the variables of the task and the solution plan used by the students in each case. From the results of this study, we derive conclusions regarding the characterisation of this kind of modelling task and the potential use of this sequence to promote problem solving flexibility.

Keywords Context variables · Estimation · Modelling · Pre-service teacher · Task variables · Solution plan

Some detail related to study' design

Primary school pre-service teachers | Strategies (models)

- We relied on the notion of "task variable" by Kilpatrick (1978), in our case we will focus on *context variable* understood as the physical characteristic of the elements of the real context that can influence solving process. In the case of 2-dimensions real context estimation tasks, the context variables are:
  - size of the elements: big (more than 1 m2), medium (between 1 cm2 and 1 m2) or small (less than 1cm2);
  - size of the area: big (about 1002 m2), medium (about 102 m2) or small (about 12 m2);
  - shape of the elements: homogeneous or heterogeneous;
  - distribution of the elements: there's a regular pattern, there's no regular pattern;
  - shape of the enclosure: the enclosure can be a simple shape (rectangular, triangular, …) or it can be the combination of different simple shapes.



Rectangular

Sequence of tasks

P1-People. How many students can stand on the faculty porch when it rains?
P2-Tiles. How many tiles are there between the education faculty building and the gym?
P3-Grass. How many blades of grass are there in this space?
P4-Cars. How many cars can fit in the faculty parking?

Problem	Element size	Area size	Element shape	Distribution of elements
P1	Medium	Medium	Heterogeneous	Non regular
P2	Medium	Medium	Homogeneous	Regular
P3	Little	Little	Heterogeneous	Non regular
P4	Large	Large	Homogeneous	Regular

Data analysis

First we qualitatively analysed the written production using the following categorization of strategies:

- Incomplete resolution: is the one where not enough detail to obtain the estimate is given.
- Counting: In this case, students just propose a direct exhaustive counting procedure to get the estimation.
- Linearisation: This corresponds to the productions that propose a one-dimensional emerging model.
- Base unit: students proposed, from one side, to obtain the total area of the rectangle, and from the other side, to obtain the estimate by dividing this area by the area occupied by an element.
- Density: students proposed, from one side, to obtain the total area of the rectangle, and from the other side, to obtain the estimate by multiplying this area by the estimated density.

Reliability: Cohen Kappa >0.77

Results

Table 10.2 Classifications and frequency of student productions for each problem ($n = 224$)

Task	Incomplete	Counting	Linearisation	2D-base unit	2D-density
P1 People	34 (15.2%)	1 (0.4%)	28 (12.5%)	110 (49.1%)	51 (22.8%)
P2 Tiles	34 (15.2%)	6 (2.7%)	92 (41.1%)	71 (31.7%)	21 (9.3%)
P3 Grass	44 (19.6%)	2 (0.9%)	15 (6.7%)	67 (29.9%)	96 (42.9%)
P4 Cars	28 (12.5%)	4 (1.8%)	31 (13.9%)	160 (71.4%)	1 (0.4%)
Total	140 (15.6%)	13 (1.5%)	166 (18.5%)	408 (45.5%)	169 (18.9%)

In order to determine whether there is a statistical relationship between the context variables of the problems and the categorised solution plans proposed by the students, we have performed an inferential analysis based on the Chi-Square Test for independence ($p<0.0001$), Pearson coefficient=0,511, $p=0.001$. We infer then that there is strong relationship between problem and strategy.



Relations entre contexte, situation et schéma de résolution dans les problèmes d'estimation

Irene Ferrando · Carlos Segura · Marta Pla-Castells

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The first aim of this study was to **validate the results of the previous work**. The results of the previous experience were contrasted with another experience in which the students had to solve an equivalent sequence (from the point of view of the context variables) although the problem statements had a higher degree of textual complexity.

In the second sequence, the problem statements are more complex, because the question regarding the quantity to estimate is posed (indirectly) after a contextualized situation.

Thus, the second aim of this study is to **identify differences in solvers' performance** related to the differences of textual complexity between both sequences.

Methodological issues

- In order to confirm the relationship of dependence between the context variables and the resolution strategies, we designed a sequence of 4 problems equivalent to the previous one in which, however, we changed the structure variable.
- The structure of a problem is linked to the textual complexity of the statement (Leiss et al., 2019).

P2. Bricks. How many tiles are there between the Faculty of Education building and the gymnasium?



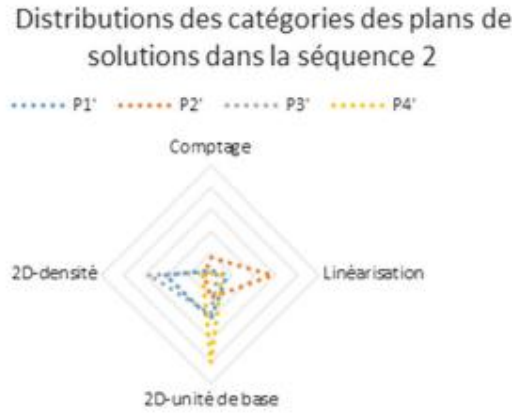
P2'. Painted bricks. The Dean's Office decided to give a touch of colour to the faculty building and therefore decided to install a scaffold on the north wall so that each student of the faculty could paint one of the grey bricks on the facade as they wished. Since each student will only be able to paint one brick, how many students will be able to participate?



Results

- The relationship between problem and strategy was confirmed also in the second sequence.
- When looking at the complete solutions (discarding the incompletes), we get that the distribution of strategies is similar in both sequences.
- When looking at the performance (measured as the ability for providing a valid strategy), we observe that the differences in performance rates and significant for the first two problems (but no for P3-P3’ and P4-P4’)





	P1-P1’	P2-P2’	P3-P3’	P4-P4’
Performance rate	84,7%-70%	84,7%-73%	80,3%-80,4%	87,5%-86,2%



Primary school pre-service teachers | Errors

Article

Classification and Analysis of Pre-Service Teachers' Errors in Solving Fermi Problems

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Abstract: Fermi problems are useful for introducing modelling in primary school classrooms, although teachers' difficulties in problem solving may hinder their successful implementation. These difficulties are associated with the modelling process, but also with the estimation and measurement skills required by Fermi problems. In this work, a specific categorization of errors for Fermi problems was established, and it allowed us to analyse the errors of N = 224 pre-service primary school teachers. The results showed that prospective teachers make a large number of errors when solving this type of task, especially conceptual ones, which are associated with the process of simplifying/structuring the real situation and the mathematization process. They also showed that there is a significant relationship between the characteristics of the problem context and the error categories. Knowing the types of errors that prospective teachers make and designing task sequences that make them emerge so that prospective teachers learn from them could be an effective way to improve initial teacher education in modelling and estimation problem solving.

The aim of this work is to study the errors made by prospective teachers when faced with solving a sequence of Fermi problems.

To do so, first of all, a categorisation of errors is established. Then, it is used to identify the existence of a relationship between the type of error committed and the contextual features of the Fermi problem.

Difficulties and errors in modelling tasks, some references

- Kaiser et al. (2010) identified difficulties in understanding the real situations to be modelled and also in determining the relationship between the real situation and the mathematical knowledge required for its solution.
- Crouch and Haines (2007) identified 4 main types of errors and difficulties in the resolution of a modelling task: inconsistency; inappropriate model; incorrect or incomplete math development; failure to validate..
- Klock & Siller (2020) developed a comprehensive categorisation that provides an overview of the difficulties at each stage of the modelling cycle.
- Moreno et al. (2021) adapted the categorization by Crouch & Haines and applied it in the analysis of the resolutions of prospective Secondary Mathematics teachers when faced with a modelling task.

Primary school pre-service teachers | Errors

Error category system in Fermi problem-solving

In order to establish this categorisation we relied on previous work on errors in modelling tasks but, in addition, we also took considered work focusing on errors in estimation and measurement tasks (Castillo-Mateo et al., 2021).

Category	Category Values	
Simplification error	E1. Incomplete initial model associated with the lack of consideration of elements of the real situation. E2. Incorrect initial model due to error of perception of the magnitude. E3. Incorrect initial model due to inadequate internalisation of referents of the magnitude to be estimated. E4. Does not build an initial model.	Conceptual
Mathematization error	E5. Mathematical model incoherent with the initial model due to an error in the meaning of the terms of the magnitude. E6. Mathematical model incoherent with the initial model due to inadequate internalisation of units of measurement of the S.I. of the magnitude to be estimated. E7. Mathematical model incoherent with the initial model due to the use of unsuitable units of measurement. E8. Mathematical model is not constructed or is incomplete because elements of the initial model are not quantified.	Conceptual
Mathematical working error	E9. Use of incorrect calculation procedures or calculation errors. E10. Error in conversion of measurement units. E11. Incomplete resolution.	Procedural
Interpretation error	E12. Absence of measurement units in the results. E13. The estimate is clearly incompatible with the real situation.	Conceptual

Results (relationship between problem' context features and errors)

- We observe that there is a dependence (weak) between the problems and the types of errors committed.
- FPs are accessible problems because a low number of them remain incomplete, but the high number of errors reflect shotcomings in the modelling competence and in the estimation and measurement skills of prospective teachers.
- During the process of simplifying and structuring the situation of the Fermi problem, most errors were related to shortcomings in the sense of estimating surface measurements.
- During mathematisation, the most frequent errors corresponded to erroneous perception of the magnitude, and are also due to leaving the mathematical model unfinished.
- In the mathematical work phase, procedural errors were numerous, although fewer than in the two previous phases. The most frequent is inversion error.
- There were few interpretation errors.



Hopefully, but not yet accepted…

Primary school pre-service teachers	Errors	Strategies (models)	Performance	Flexibility

Pre-service teachers' flexibility and performance solving Fermi problems

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Fermi problems allow the introduction of modelling in primary school education, and their solution requires the development of quantity measurement skills. To ensure their effective introduction in the classroom, teachers must have adequate proficiency to deal with them. One of the key aspects of problem-solving proficiency is flexibility, but there are few studies on flexibility in solving real-context problems. This study, based on an analysis of the errors made by 224 prospective teachers when solving a Fermi problems sequence, establishes performance levels. In addition, we define levels of flexibility in using multiple solutions across the sequence, which allows us to address the main objective: to study the relationship between performance and flexibility. We found that there are significant relationships between flexibility levels and the number and severity of errors made. Encouraging flexibility in prospective teachers may be an efficient way to improve their performance in solving real-context problems.

The aim of these study is to relate all the previous results with the notion of flexibility.

We know that the prospective teachers provide different solutions to similar problems depending on their characteristics, then, using the FPs sequences our aims are:

- Analysing performance
- Analysing flexibility
- Searching for possible relationship between performance and flexibility

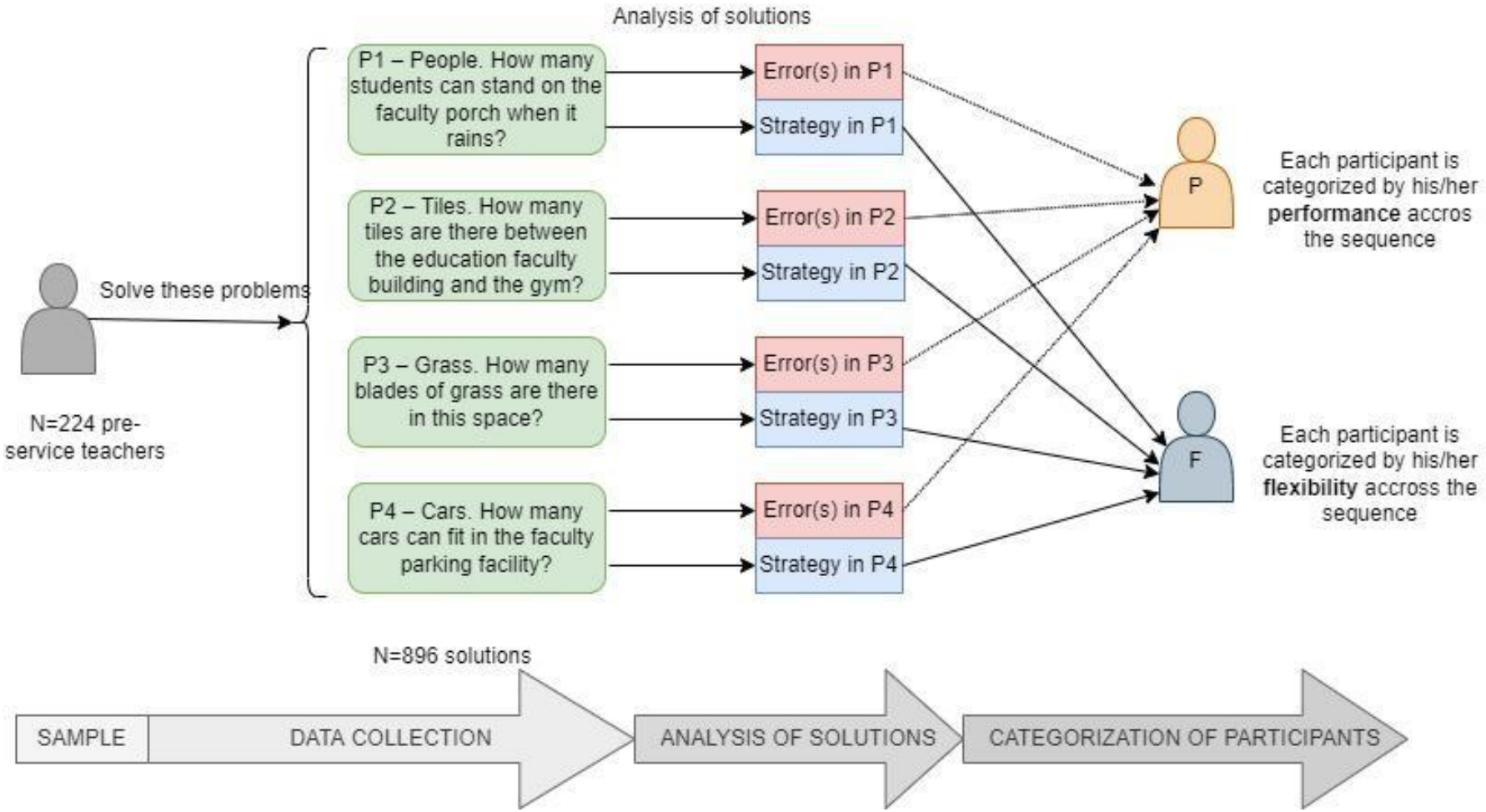
Flexibility in problem-solving

- Flexible strategy use refers to individuals being able to choose between different solution strategies when dealing with a mathematical activity. Flexibility is considered an important mathematical skill (Heinze et al., 2009).
- Most studies on the influence of the development of multiple solutions strategies and their flexible use have focused on intra-mathematical tasks some relate flexibility with performance. But there are few empirical studies linking flexibility and performance in real-world problem-solving (Schukajlow & Krug, 2014; Schukajlow et al., 2015; Achmetli et al., 2019).
- We consider that a solver shows flexibility across a FPs sequence when switching strategies in any of them (Elia et al. 2009). We will study the relationship of this type of flexibility and prospective teachers' performance in problem-solving.

Prospective teachers' problem solving performance

- Chapman (2015) highlights the importance of teacher knowledge of problem-solving.
- Studying errors made by pre-service teachers when solving real-context problems can be useful to measure their performance level (Klock & Siller, 2020).
- In the previous study we established a system of errors specific to FPs, within this system of errors, it is possible to differentiate between errors that impede devising a valid strategy (strategic errors) and mathematical errors. Based on Copur-Gencturk and Doleck (2021) approach, the analysis of the errors committed throughout a sequence of Fermi problems will allow us to define three levels of performance.

Methodological design of this study



Categorization of performance and flexibility across the FPs sequence

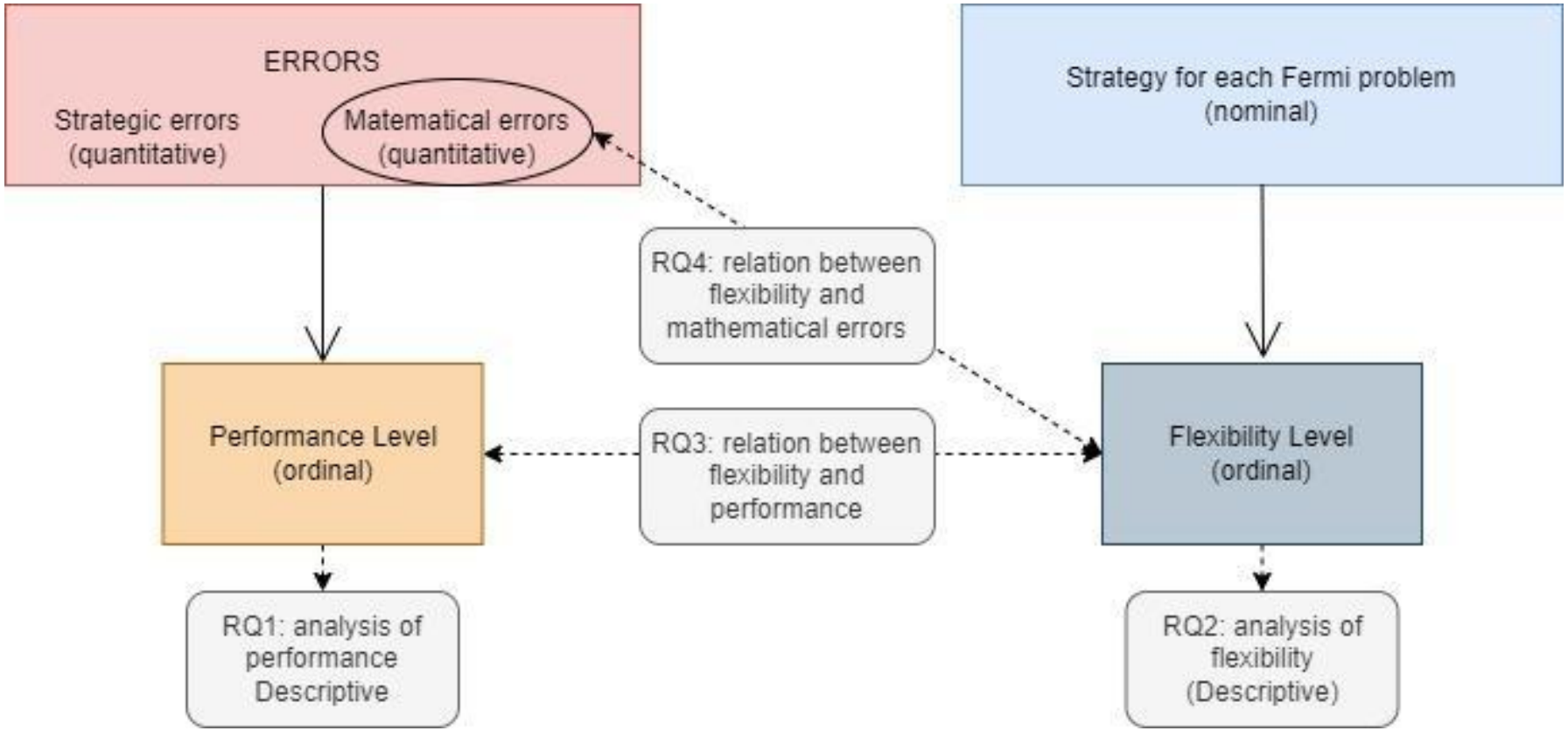
Performance:

- LOW (level 0) when the participant proposes at least one incomplete solution.
- BASIC (level 1) when the participant does not make errors that impede devising a valid strategy in any of the solutions, but there is at least one solution with a mathematical error.
- HIGH (level 2) when the participant makes no errors in any of the solutions to the four tasks.

Flexibility:

- Non-flexible. Includes prospective teachers who proposed the same solution strategy in all completed tasks.
- Moderately flexible. Includes those who proposed two different valid solution strategies, but switched solution strategy only on one problem.
- Very flexible. Includes those who proposed two or more different valid solution strategies and switched solution strategy in two or more problems.

Description of variables and research questions



Results

- Results have shown that three fourths of the pre-service teachers who participated in this study made errors when solving Fermi problems.
- Concerning flexibility, more than two thirds of the participants were categorized as (moderately or very) flexible.
- Flexibility and performance have been found to be related; in fact, when studying in detail the number of mathematical errors made, it was found that flexibility was associated with a reduction in such errors.

Limitations and conclusions

- This is a correlational study, so we cannot determine dependencies between variables. It is necessary to deepen this research by designing other studies that help us to determine whether a flexibility influences performance or whether performance that influence flexibility.
- Since studies focused on the use of multiple solving strategies for real-context and modelling problems are scarce, this work contributes a novel approach, using sequences of Fermi problems to study flexibility.
- Flexibility is one of the criteria for evaluating creativity (Levav-Waynberg & Leikin, 2012), and recently Lu and Kaiser (2022) have enriched modelling competencies by including creativity. A future line of research is to deepen dependencies between flexibility and performance, which would help to better understand creativity in solving modelling problems, such as Fermi problems.

Thanks! Danke! ¡Gracias!

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Análisis de los factores de complejidad en planes de resolución individuales y resoluciones grupales de problemas de estimación de contexto real

Analysis of the complexity factors in individual solution plans and group developed solutions for real context estimation problems

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Primary school pre-service teachers

Complexity elements

In this study we analyse the productions of prospective teachers (same experience as previous studies) in order to identify the so-called “complexity elements” (similar to the notion of “realistic assumption” by Krawitz).

We compare individual and in situ group work solution.

Analysis of complexity elements

For the analysis we relied on the solution scheme obtained in Albarración et al. 2020:

- Obstacle removal
- Average size/density
- Differentiated sizes/densities

Results

- We observe that only 41% of participants include complexity elements in their individual solutions to the FPs of the sequence, while this rate increases to 83% in their group solutions.
- We should bear in mind that group solutions are done in situ, and, of course, this can be the reason for that difference. A limitation of that study is that we the design of the experience is not good enough to identify the factor that fosters the introduction of realistic assumptions.
- We also identify some relations between the context features and the complexity elements considered.