CERME 13

13TH CONGRESS OF THE EUROPEAN SOCIETY FOR RESEARCH IN MATHEMATICS EDUCATION

Relationship between strategies and appropriateness criteria for Fermi problems.

An exploratory study with expert mathematics teachers

Carlos Segura & Irene Ferrando

Mathematics Education Department, Universitat de València (Spain)

The authors are grateful to the grant PID2020-117395RB-I00 and PID2021-126707NB-I00 funded by MCIN/AEI/ 10.13039/501100011033 and by "ERDF A way of making Europe".

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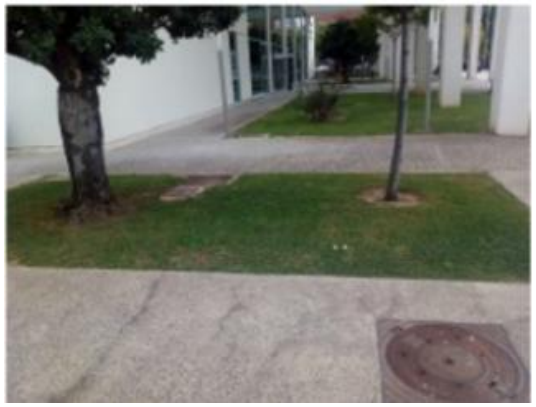
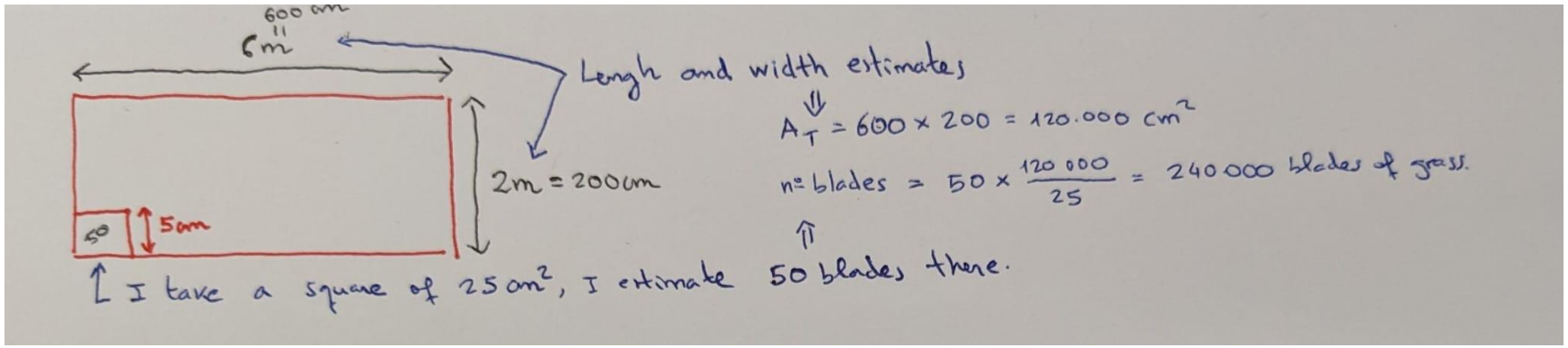
Comparison of multiple strategies for Fermi problems

Which strategy is better to solve the following 2D Fermi problem?

P3- Grass. How many blades of grass are there in this space?

Real, open-ended problems require students to make assumptions about the situation to reach an estimate (Ärlebäck, 2009).

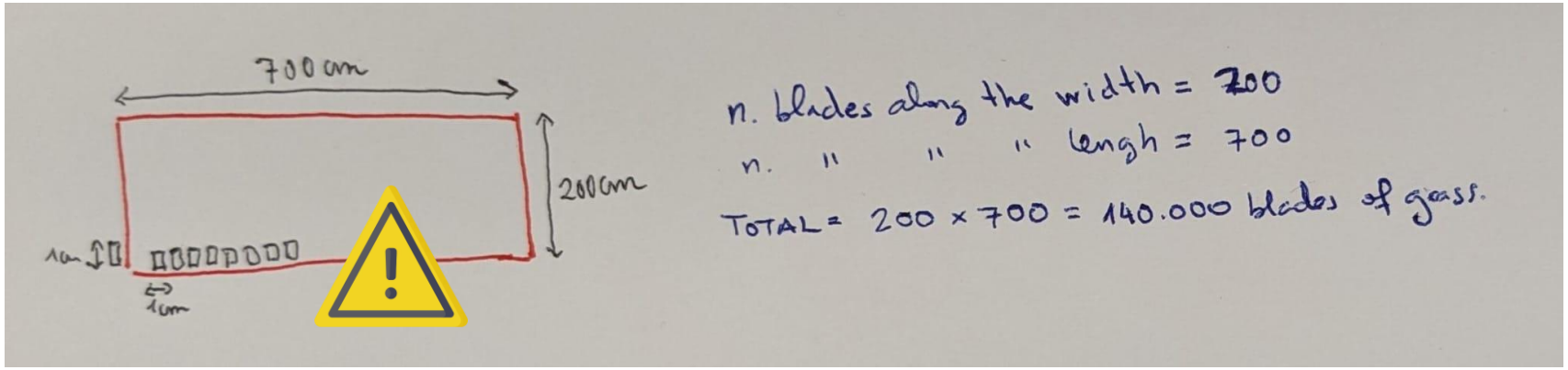




DENSITY



LINEALISATION



Adaptivity in solving Fermi problems?

- **Adaptivity:** Selection of the most appropriate strategy according to the characteristics of the problem (Verschaffel et al., 2009).
- It is a theoretical challenge (Heinze et al., 2009). What *criteria* can we use (and how) in Fermi problems?
  - **Rapidity** (fewer steps) and **accuracy** (Lemaire & Siegler, 1995).
  - For real problems, previous work (Albarracín et al., 2021, Segura et al., 2021) shows that the strategy can be more careful, incorporating real assumptions or empirical procedures, which we will call **rigour**.

An exploratory study with 81 experts mathematics teachers (experience with Fermi problems)

RQ1. Which strategies are the most appropriate to solve a Fermi problem according to its contextual features?

RQ2. What criteria justify the choice of a certain strategy as the most appropriate?

- Four sections, each section devoted to a 2D Fermi problem (with different contextual features).
- Two closed questions for each problem:

  W*hich of the following strategies is the most appropriate for this problem?* [C, L, BU, D, N].

  Which of the following criteria did you use to choose the most appropriate strategy? [rapidity, accuracy, rigour, other].

Adaptivity in solving Fermi problems?

How many students can fit on the porch of the Faculty of Education?



Which of the following strategies is the most appropriate for this problem? *

- ◯ Strategy 1. Counting. Count the students entering the porch.
- ◯ Strategy 2. Linearisation. Estimate the number of students that fit the length of the porch and multiply by the number of rows that cover the width to get the number of students.
- ◯ Strategy 3. Base unit. Compute the area of the porch and the area occupied by one student and divide the total area by the area occupied by one student to obtain the number of students.
- ◯ Strategy 4. Density. Determine a unit area within the porch and estimate how many students fit in it. Obtain the density (no. students/unit area). Calculate the total area of the porch in area units. Finally, multiply the density by the total area of the porch to obtain the number of students.
- ◯ There is no one strategy that is more appropriate than another.

RQ1 Results

- There is a significant association between problems (contextual features) and strategies: $\chi^2 = 108.88$, $LR = 142.86$ with $p < 0.001$. Medium effect ($V = 0.34$).
- *P1-People* (**medium size elements, disordered**) **Base unit** (43%) and **Density** (42%) are chosen as the best strategy.
- *P2-Pavers* and *P4-Cars* (**ordered distribution of elements**) **Base unit** (49%/37%) and **Linearisation** (25%/32%) are chosen.
- *P3- Grass* (**small and irregular size of elements, very disordered**) hegemony of **Density** (83%).

Adaptivity in solving Fermi problems?

RQ2 Results

	Counting	Linearisation	Base unit	Density	No more appropriate
Accuracy	5	**29**	**44**	**78**	4
Rapidity	0	8	**78**	34	4
Rigour	1	**35**	17	**72**	1
No criteria	0	0	0	2	11

Strong association between strategies and criteria: χ^2 (12, 423) = 276.53 (*LR* = 153.74) with $p < 0.0001$ and $V = 0.47$).

Table 3: Most appropriate strategies for each problem and their criteria

	Counting	Linearisation	Base Unit	Density
P1			Rapidity	Rigour
P2		Rigour	Rapidity	
P3				Rigour
P4		Rigour	Rapidity	

- **Accuracy** is a potential criterion (Hickendorff, 2022) for all strategies, not a practical one (we do not know the solution to validate estimates).
- **Rapidity** (smaller number of steps and calculations) is linked to **Base Unit**.
- **Rigour** is linked to **Density**, and also **Linearisation** (but only in "ordered problems": P4-*cars, P2-pavers*).
  - ➢ Because Density and Linearisation relies on partial estimations, giving "reliabilty" to the process.
- Putting RQ1 and RQ2 together: "best" strategy according to a criteria for each problem.

Thank you very much for your attention!

carlos.segura@uv.es

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