PROBLEM POSING AND PROBLEM SOLVING BY PROSPECTIVE TEACHERS: AN ANALYSIS OF THE INFLUENCE OF THE CONTEXT

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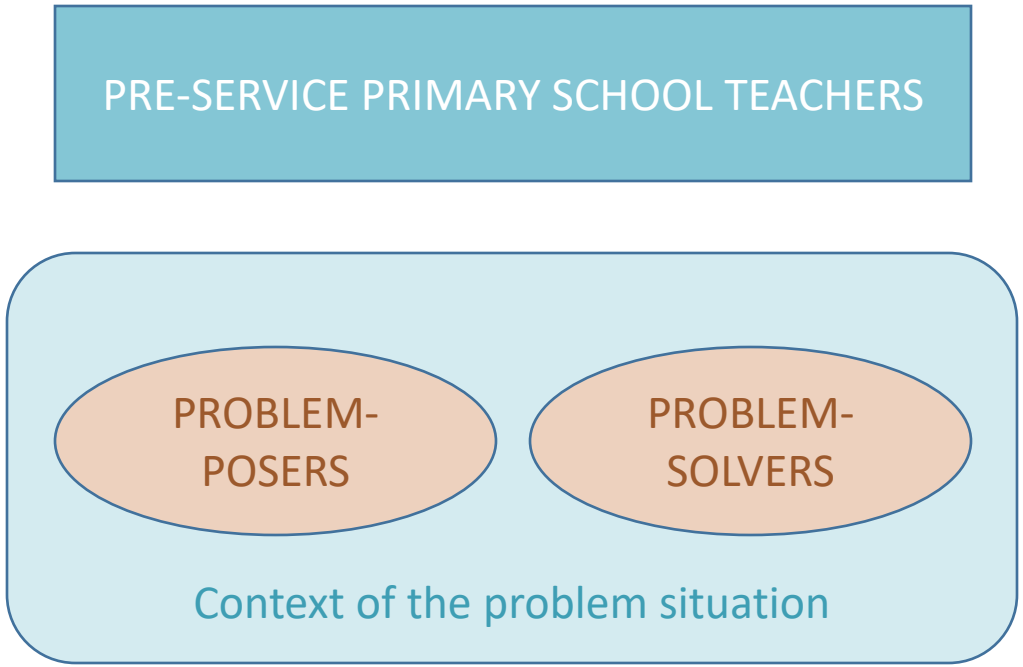
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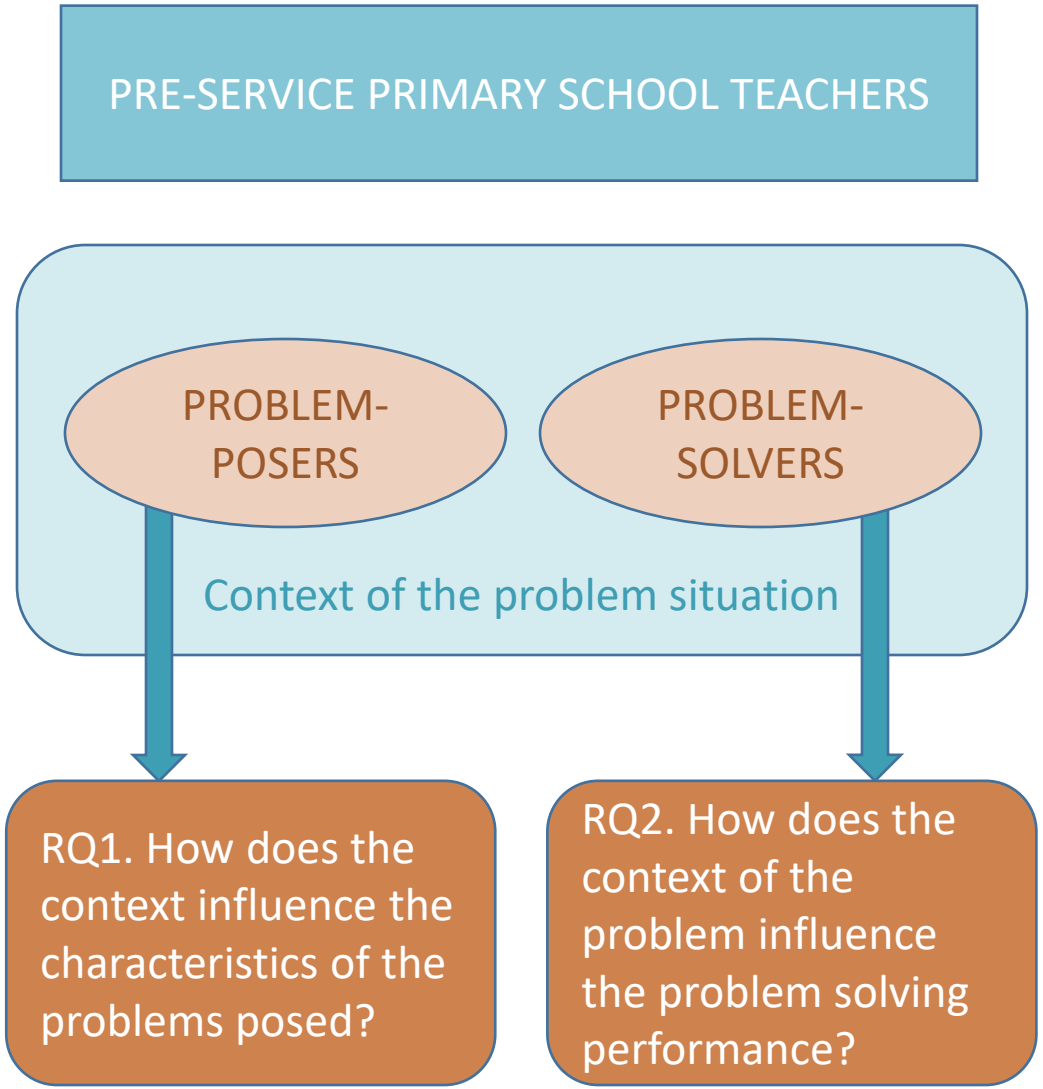
PME-46
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Framework & Motivation

- New Spanish educational program emphasizes the importance of problem solving and problem posing in primary school. Moreover the use of different (real) contexts is highlighted.
- *To teach for Problem Solving (PS) proficiency, teachers should be proficient in PS and understand the nature of it in order to teach it effectively* (Chapman, 2015, p.24).
- When a problem-posing activity is undertaken, the context influences the characteristics of the problems posed (Baumanns & Rotts, 2021; Hartmann et al., 2021). The context can influence problem-solving performance when dealing with estimations problems (Albarracín et al., 2021).



Definitions & Research questions



- Here *problem posing* refers to the process of developing new problems.
- For stimulating problem posing different situations or prompts can be used (Cai et al. 2022) → Here prompts will be *unstructured* (Baumanns & Rott, 2021) and will include descriptions of situations and/or pictures.
- Problems can be categorised according to their *relation to reality and also to de degree of familiarity of the real context*, the same happens for problem solving situations (Hartmann et al., 2023).
- When analysing problem solving performance of real context estimation tasks it is possible to differentiate *performance level according* to the development of the solution (Segura & Ferrando, 2023)

Methodology: Design

For the design of the experience we were inspired by the research conducted by Hartmann et al. (2021).

Phase 1. Problem posing

Below, you will find three situations with different characteristics. Unlike what you are used to in maths lessons, we are not going to ask you to solve any maths problem, but you will be the one who poses the problem. Start by reading and looking carefully at each of the situations. Once you have read and looked carefully at each of the three situations, think about a math problem that you can pose from each situation and write down the statement clearly.

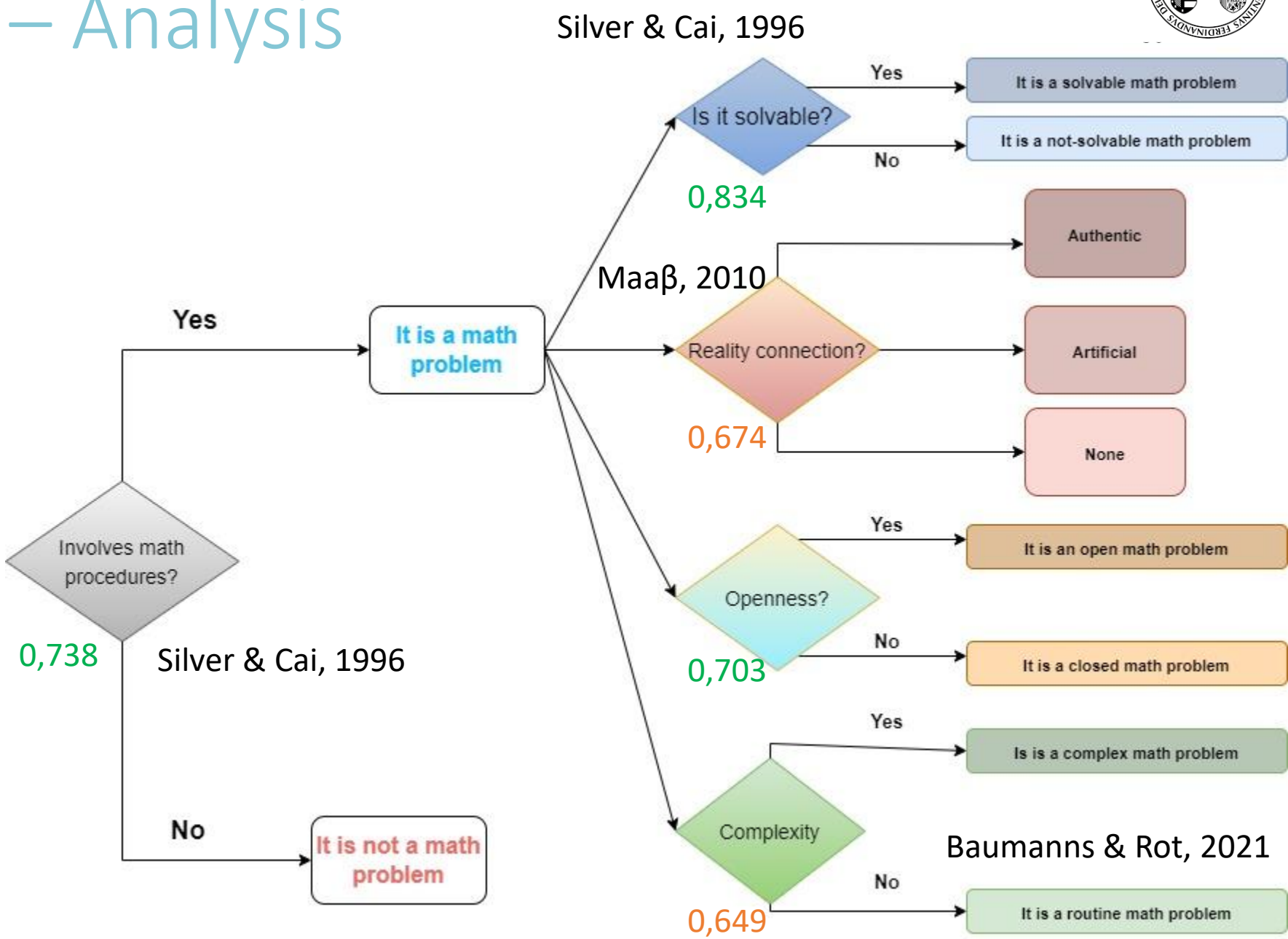
Phase 2. Problem solving

Methodology – Participants & development

- 106 primary school pre-service teachers (74% women, average age 20,4)
- Individual work
- Phase 1: we provided the participants with the 3 situations and gave them 1 week to pose, at least, one problem for each situation
- Phase 2: during a 45 min session the participants should solve each problem

Methodology – Analysis

- Mixed analysis: for the qualitative analysis of posed problem we followed this coding scheme.



Methodology – Analysis

- Mixed analysis: for the qualitative analysis of **problem solving performance** we followed this coding scheme (inspired by Hartmann et al., 2021), we give 0 or 1 to each element:

Solution strategy (math model) **M**	Math result **R**	Interpretation **I**	Realistic assumption **A**
0,754	0,692	0,827	0,651

$$Performance = 3 \times M + R + 2 \times I$$

Results about characteristics of **posed problems** (I)

	Not math problem	Non solvable math problem	Solvable math problem
Real close	10 (9,43 %)	10 (9,43 %)	86 (81,14 %)
Real far	1 (0,01 %)	5 (0,05 %)	100 (99,94 %)
Not real	55 (51,89 %)	17 (16,04 %)	34 (32,07 %)
TOTAL	66 (20,15 %)	32 (10,06 %)	220 (69,18 %)

The participating prospective teachers have more difficulties in posing mathematical problems based on a non-real situation ($z=90,979$, $p<.001$)

The future teachers participating in the study were able to pose a significantly higher number of solvable problems from the context away from their own reality than from the context close to them ($z=7,403$, $p=,003$)

Results about characteristics of **posed problems** (II)

When confronted with a **close real situation**, most of the participants pose a problem artificially connected to reality (59,4%) that are closed (60,4%) an routine (61,5%)

When confronted with a **far away real situation**, most of the participants pose a problem artificially connected to reality (49,5%) that are closed (97,1%) an routine (89,5%)

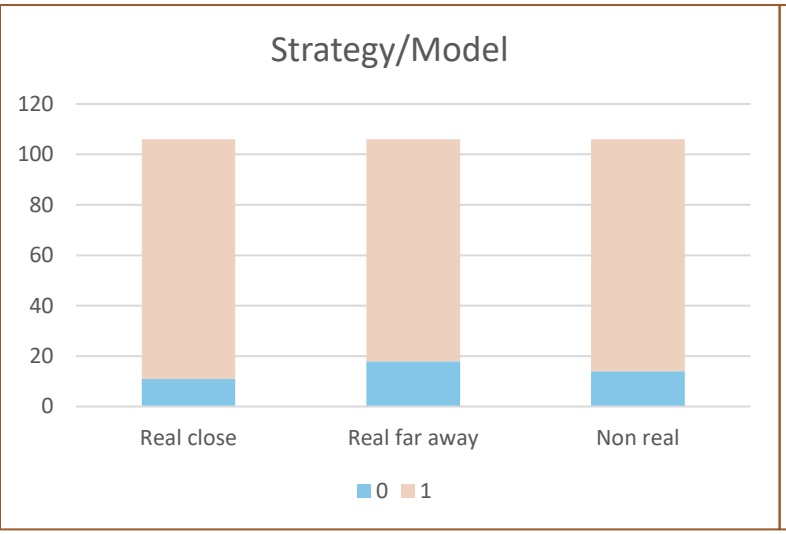
When confronted with a **non real situation**, most of the participants pose a problem NOT connected to reality (84,3%) that are closed (98%) an routine (88,2%)

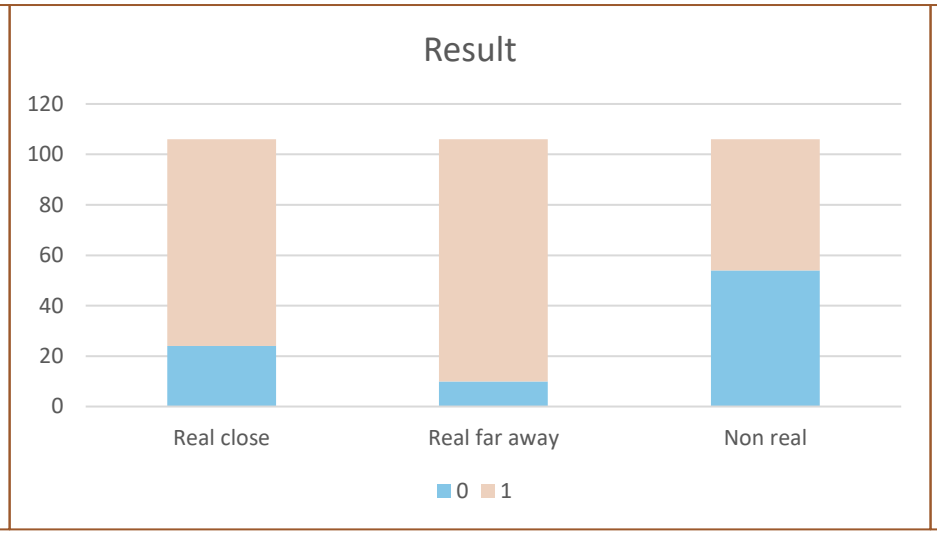
CONNECTION TO REALITY: Both real contexts foster participants to pose real context problems, but **the connection to reality is mainly artificial (particularly in real close context)**

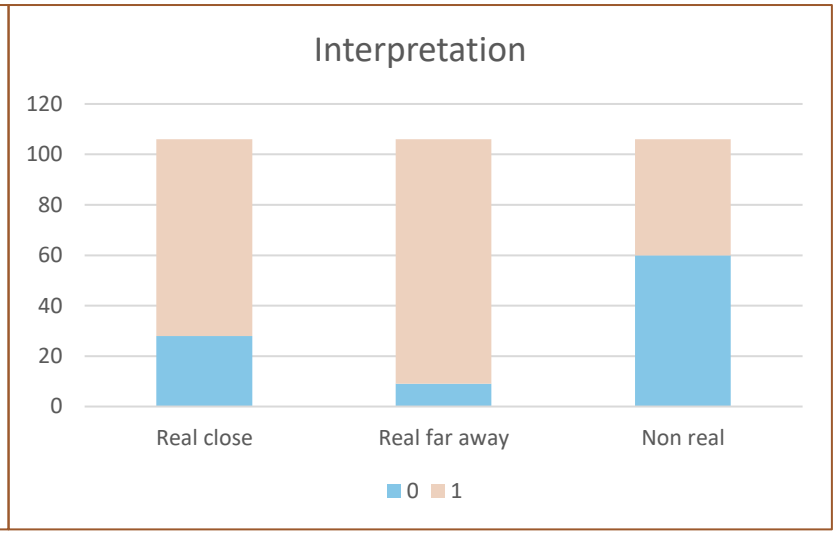
OPENNESS: We find that, while in general participants pose more closed problems, **the close real context fosters participants to pose significantly more open-ended problems** ($Z=75,612$, $p<.001$).

ROUTINE: While overall more than 78% of the problems posed from the scenarios are routine problems, **the proportion of non-routine problems posed from a close real context is significantly higher than in the other two contexts** ($Z=26,96$, $p<.001$)

Results about **problem solving** performance







Overall, 87% of the resolutions include a correct approach, although the second problem is the one that generates the most difficulties at this stage.

While overall the proportion of correct resolutions is quite high (72% of the total), the proportion of incorrect results in the case of the problem with no connection to reality is significantly higher than in the other two contexts ($Z=42,6$, $p<.001$)

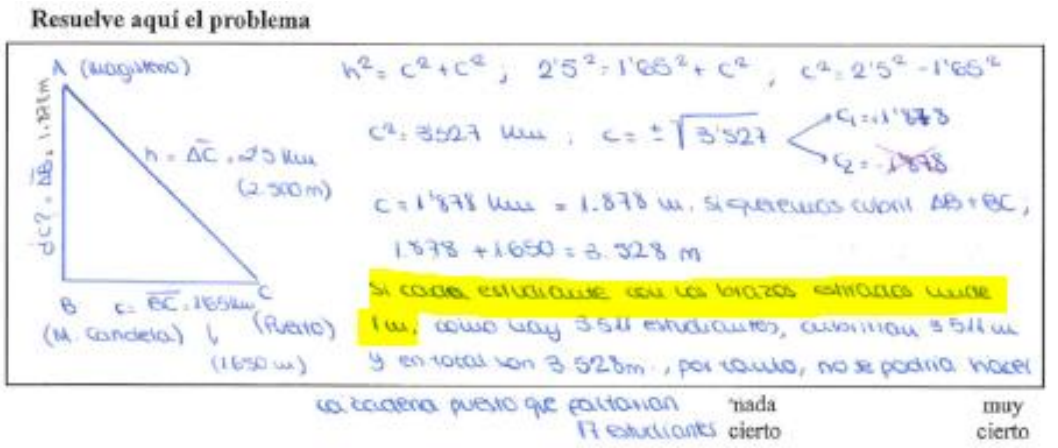
We observe that in the third problem (without real context) the proportion of correct interpretations is significantly lower than in the other two problems.

Results about **problem solving** performance

	0	1	2	3	4	5	6	Average (SD)
Real close	2	2	4	7	20	14	57	4,93(1,45)
Real far	1	3	0	16	3	7	76	5,23(1,42)
Not real	9	1	0	45	9	4	38	3,96(1,84)

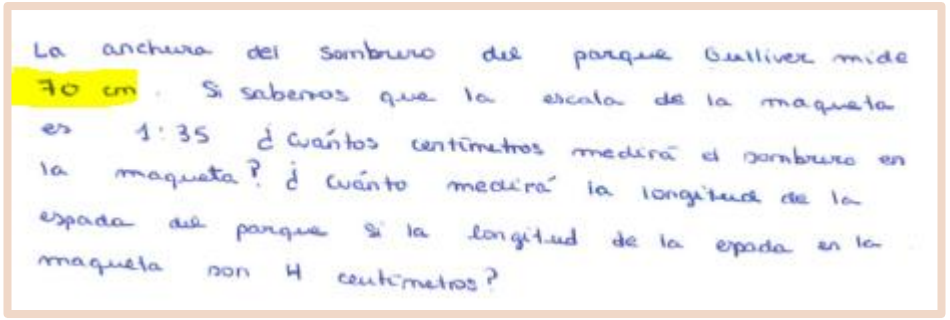
We observe that there is a dependence relationship between the context of the problem and performance (Fischer test general and paired)

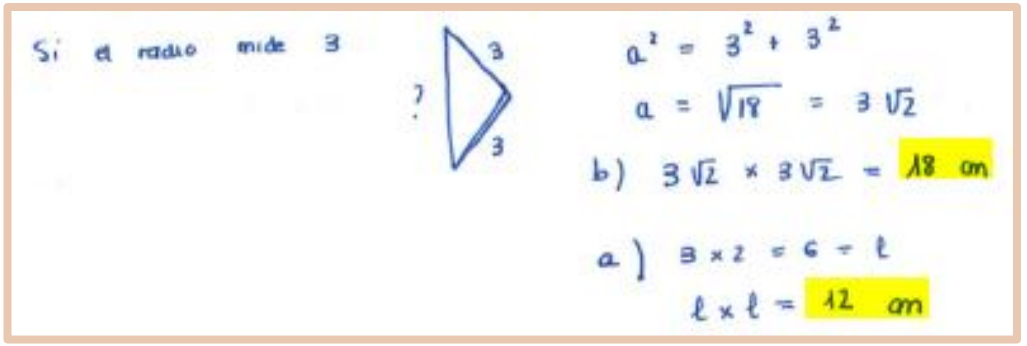
	Without realistic assumption	With realistic assumption
Real close	31 (29,2%)	75 (70,8%)
Real far	104 (98,1%)	2 (7,9%)
TOTAL	135 (63,7%)	77 (36,3%)



Discussion & Conclusions

- Our results about problem posing are aligned with Silver & Cai (1996) and Leung & Silver (1997): most of the posed problems are mathematical solvable problems (closed). The future pre-service teachers participating in that study tend not to pose modelling problems.
- When confronted with a real context, some pre-service teachers tend to pose problems artificially related to reality, this happens, in most of the cases, because they have difficulties with measurement estimation (Jones et al., 2012).
- In performance analysis, we observe that participants have difficulties when interpreting the information provided in the problem statement (particularly in real context).Tha is difficulty appears also when interpreting the solution (particularly in the third problem), because they are frequently not able to generalize to give an answer. Real close context fosters the incorporation of realistic assumptions.





LIMITATIONS: the sample is not big enough (exploratory study),performance analysis can be improved identifying levels through error analysis (as in Segura& Ferrando, 2023), the results are strongly dependent on the situations and problems, it would be interesting to analyse the development of the phases in posing and solving (as done by Hartmann et al., 2023)

¡Gracias! Thanks! !תודה

Please, don’t hesitate to write me for further comments, recommendations, or ideas ☺

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Methodology: Design. Phase 1 problem posing

Seguro que todos conocéis el parque Gulliver que está en el parque del Turia, un poco antes de llegar a la zona de la ciudad de las artes y las ciencias, pero igual no sabéis algunos aspectos interesantes… El artífice de la idea original fue el arquitecto municipal Rafael Rivera, pero ningún escultor se atrevía a materializarla. El artista fallero Manolo Martín se entusiasmó de inmediato con la idea y el proyecto empezó a tomar forma. Se incorporó al proyecto un dibujante de cómics, Sento Llobell, que hizo una primera propuesta.





Todo el saber del arte fallero se puso al servicio de la arquitectura, la figura de Gulliver se construyó en el taller de Manolo Martín con la colaboración de numerosos artistas falleros. Se trabajó con una maqueta a escala 1:35 partida en 68 rodajas.

Actualmente el parque está cerrado debido a una obra integral de rehabilitación y reabrirá a finales de este mismo año.

Gulliver Park. Real situation, close context

En la Edad Media, la sal se obtenía de la evaporación del agua del mar. Hoy en día se obtiene principalmente mediante la minería. La sal (1,2 t por m3) se apila en montañas de sal usando cintas transportados de 1,2 m de ancho.

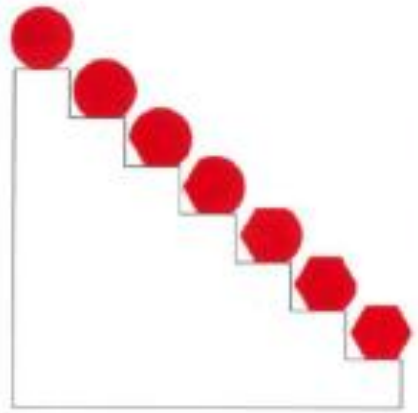
En la figura puedes ver una montaña de sal. La longitud de la ladera es c=20m y el diámetro es d=30m. La montaña de sal pesa unas 3740 toneladas.

Datos técnicos del camión de transporte:

- Plataforma de carga del camión: 5,11mx2,3mx1,8m
- Potencia: 125 kw
- Capacidad de volumen: 5249 cm3
- Carga máxima: 26,8 toneladas



Salt Mountain. Real situation, far context



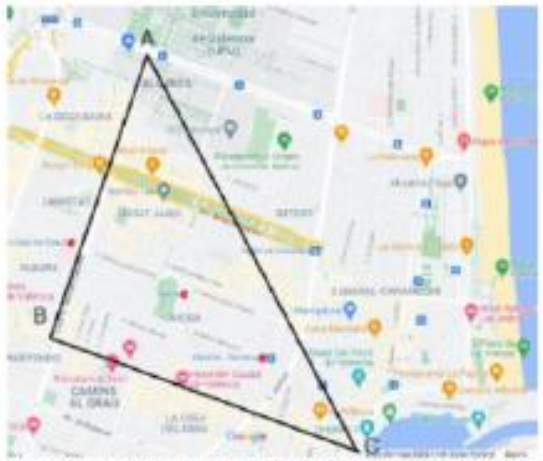
Aquí al lado, te muestro un poema visual de poeta Toni Prat. En un poema visual hay, en la mayoría de casos, un elemento icónico a través del cual se define el poema.

Fíjate detenidamente en el poema que, como ves, tiene una componente matemática importante.

Non real situation

Methodology: Design. Phase 2 problem solving

Como sabéis, el Ayuntamiento de Valencia ha previsto una ampliación del puerto que permitirá un crecimiento de sector de importación y exportación de mercancías pero que, según diferentes informes, también conllevará daños ecológicos serios a medio y largo plazo. El alumnado de la Facultat de Magisteri, organizado a través de los sindicatos de estudiantes, ha organizado una movilización para llamar la atención de la ciudadanía sobre los riesgos de la ampliación del puerto de Valencia.



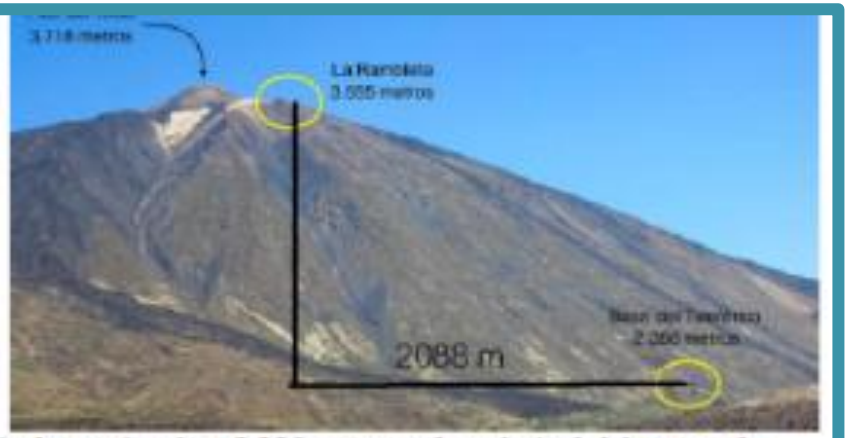
Para ello, se va a realizar una cadena humana que unirá la Facultat de Magisteri con la zona norte del puerto a través de dos avenidas de gran afluencia: la avenida Manuel Candela y la Avenida del Puerto. Así, la cadena humana cubrirá los lados AB y BC de triángulo que puede ver en la siguiente imagen.

Sabiendo que en la Facultat de Magisteri hay un total de 3511 estudiantes matriculados en uno de los dos grados, que la distancia del recorrido en Manuel Candela es de 1,65km y que la distancia en línea recta desde la Facultat de Magisteri (A) hasta el puerto (C) es 2,5 km, trata de averiguar si será posible formar una cadena humana con el alumnado participante en la movilización.

Human chain. Real situation, close context

El cable del teleférico del Teide debe ser remplazado. Cada metro de cable tiene un coste de 8€. ¿Qué precio aproximado tendrá el remplazamiento del cable actual? Explica con detalle tu resolución.

El teleférico del Teide se encuentra en el parque nacional del Teide en Tenerife (Canarias), siendo el teleférico más alto de España. Asciende desde la estación base, situada a 2.356 metros sobre el nivel del mar en la falda del Teide, hasta la estación de La Rambleta a 3.555 metros sobre el nivel del mar y a tan solo 163 m de la cima del volcán, salvando un desnivel de 1.199 m. El recorrido dura entre ocho y diez minutos a una velocidad máxima de 8 m/s. Las cabinas tienen capacidad para 44 personas.



Vulcan cable car. Real situation, far context

A partir de las dimensiones del dibujo que aparece en la poesía, trata de averiguar las dimensiones de:

a) El menor cuadrado que contenga al círculo rojo.
b) El mayor cuadrado que esté incluido en el círculo rojo.



Non real situation

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