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Influence of M&ME prior knowledge on pre-service teachers’ performance in solving Fermi problems

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Fermi problems require the solvers to make assumptions about a real situation and measure and estimate relevant quantities (Segura & Ferrando, 2021, 2023).

For example, such a problem would be to estimate *how many cars are in a large traffic jam*.

Fermi problems require relating prior M&ME knowledge to real-world phenomena, but pre-service teachers make numerous errors when they must reason about estimating and measuring areas and lengths in real and complex situations (Baturo & Nason, 1996; Segura & Ferrando, 2021, 2023).

RQ1: How is pre-service teachers’ prior knowledge of M&ME of areas and lengths?
RQ2: Are there differences in performance on Fermi problems according to pre-service teachers’ prior M&ME knowledge?

Experience: 87 pre-service teachers at the University of Valencia (Spain). 1st session, they took a *M&ME test*. 2nd session, they completed a sequence of 7 *Fermi problems related to lengths and areas*.

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Test of prior M&ME knowledge

Design: The test focuses on **area and length** estimation and measurement skills (Hildreth, 1983; Joram et al., 1998) :

14 multiple-choice questions corresponding to the perception of area and length, understanding and use of area units, having a mental image of benchmarks (Figure 1), using appropriate terms and strategies for measuring areas and perimeters, converting different area units (Figure 2), etc.

Validation: A panel of 9 experts reviewed the items in 2 rounds. In both rounds, they assessed whether each item was well formulated and appropriate for measuring the skill to which it referred, proposing changes if necessary.

Once the items had been modified*, Lawshe's Content Validity Ratio* was calculated for each item, obtaining values greater than 0.78 for all of them, which justifies their validity.

To estimate the surface area of a desk, I will take as a reference a smartphone, which occupies an area of approximately....

a) 130 cm².

b) 310 cm².

c) 220 cm².



Figure 1

How many tiles of 25 square centimetres are needed to cover a wall of 4 square metres?

a) 16 tiles.

b) 160 tiles.

c) 1600 tiles.



Figure 2

Fermi problem sequence

Design: 7 Fermi problems consisting of estimating a large number of elements along a length or enclosed in a surface. Each problem refers to a real place near the Faculty of Education and is accompanied by a picture (Figure 3).

Problem 2. How many cars can fit in a row (one after the other) along the whole length of the avenue of the Faculty of Education?



Figure 3

Results RQ1: Performance in M&ME knowledge test

The overall average performance (0-14) in the test was 8.23. Ward's hierarchical method was used to obtain three clusters with similar sizes:

- About 40% of pre-service teachers had a low level of measurement and estimation knowledge, they failed in more than 50% of M&ME skills.
- About 30% of pre-service teachers had a moderate level, they succeeded in 50%-70% of M&ME skills.
- About 30% of pre-service teachers had a high level, they succeeded in more than 70% of M&ME skills.

Repeated measures ANOVA: Pre-service teachers' performance in items related to the area unit (understanding, use and conversion) is significantly lower.
On the contrary, in items related to applying appropriate procedures and formulae to measure areas or perimeters their performance is significantly higher.

Results RQ2: Performance in Fermi problems

We scored each Fermi problem from 0 points to 2 points:

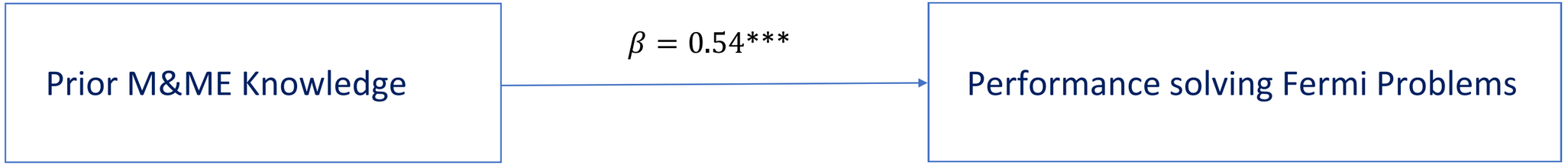
If pre-service teachers made strategic development errors on a problem they were scored 0; if they made mathematical work errors they were scored 1; and if they made no errors they were scored 2.

The average performance in the 7 FP (0-14) was 9.4.

Level of measurement and estimation skills (prior test)	N	Performance in Fermi problems
Low	37	7.38
Moderate	26	9.62
High	24	12.29

One-way ANOVA: **there are significant differences in performance on Fermi problems depending on pre-service teachers' prior measurement and estimation skills** ($F = 17.53$ with $p < 0.001$ and large effect size in the differences, $\eta^2 = 0.3$).

Post-hoc test indicates that pre-service teachers with high prior measurement and estimation skills perform significantly **better** than those with moderate ($p < 0.01$) and low ($p < 0.001$) levels. Pre-service teachers with moderate level also perform significantly better than those with low level ($p < 0.05$).



*** $p < .001$ and $R^2 = 0.29$.

We formulate a linear regression model to obtain to what extent prior M&ME knowledge explains changes in performance:

F (1, 85) = 35.16, R^2 = 0.29, β = 0.54, t = 5.93, $p < .001$.

Pre-service teachers' prior M&ME knowledge has a significant positive effect and explains 29% of the variance in Fermi problem-solving performance.

Conclusion: This study confirms the importance of prior estimation and measurement knowledge for the successful resolution of Fermi problems, as their acquisition ensures that fewer mathematical and strategic errors are made (Segura & Ferrando, 2023). Findings can help improve prospective teachers' instruction on the application of measurement knowledge in real-world situations.

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